Measuring Firm Uncertainty*

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Abstract

I propose a novel firm-level uncertainty measure based on the latent conditional volatility of forecast errors. By applying this measure to I/B/E/S database, I track 1,916 U.S. public companies' uncertainties for over 34 years. Those firm-level measurements are aggregated into a macro index and their implications at macro- and micro-level are compared. At the macro level, VAR results indicate a strong "granular origin" of uncertainty impacts from large firms. At the firm level, panel regression results confirm the negative impact of macro uncertainty and reveal a composite effect of idiosyncratic uncertainty depending on investment horizon, firm profitability, and the magnitude of shock. Furthermore, a model is used to interpret these findings.

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1 Introduction

Despite that efforts have been put into building a reliable uncertainty index, current measures are still far from perfect. One challenge is that there is no consensus on the definition of uncertainty and literature seems to be lacking such discussions. The other challenge, which seems to be more prevailing, is that uncertainty is unobservable and its measure relies on proxies. However, the translation is not always direct. For instance, the popular uncertainty index VIX is entirely based on the famous Black-Scholes model does not capture the market behaviors with "volatility smile". [1] [2] [3] Qualitative measures such as Economic Policy Uncertainty (EPU) (see Baker, Bloom, and Davis (2016)) lack innovative statistics to effectively summarize text data. Disagreement depends too much on human' prior beliefs which is not related to a specific economic event being measured. (see Zarnowitz and Lambros (1987), Lahiri and Sheng (2010), Krüger and Nolte (2016)). Any cross-sectional variance measure of companies' earnings, productivity, etc., contains predictable components that are not uncertain (see Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)). Methods such as the dispersion of density forecast and entropy are to large extent limited by the availability of compatible data (see Liu and Sheng (2018) and Istrefi and Mouabbi (2017)). Beside those challenges, there are also area to be explored: current uncertainty indices are usually constructed from macro data with little micro-level information. Campbell, Lettau, Malkiel, and Xu (2001), Bachmann and Bayer (2013) and Bijapur (2015) are some of the very few papers that addresses both. However, businesses and researchers show increasing interests in monitoring uncertainty and understanding its impacts at micro level, which requires micro measurements. To address this gap, I propose a firm-level uncertainty measure that has the following merits: (1) It is closely linked with uncertainty by its definition; (2) It could be aggregated into a macro level index; (3) The measure can be used on data covering a large number of firms for a long history.

The measure proposed in this paper focuses on the objective side of uncertainty and defines un-

[1] Higher implied volatilities for in the money or out of the money options based on the lognormal distribution assumption in the process. For details see Hull and White (1998).

[2] VIX is a measure of the implied volatility in U.S. stock market based on S&P 500 index options. It is maintained and published by the Chicago Board Options Exchange (CBOE).

[3] In addition, VIX has long been criticized for containing volatilities directly from trading (See Fama (1965)).

certainty as the conditional volatility of an unpredictable event. [4] Such a volatility is a composite result of stochastic factors in both economic phenomena and corresponding human forecasting activities. In addition, it should not be confused with the volatility of the event itself since predictable volatilities are not "uncertain" (see Jurado, Ludvigson, and Ng (2015)). In other words, I emphasize volatilities associated with unpredictable factors that lead to real economic problems. The measure is formalized as

$$e_{i,t} = Y_{i,t} - E(Y_{i,t}|I_{t-1}), \quad e_{i,t} \sim \mathcal{N}(\mu_t,\, \sigma^2_{i,t}), \tag{1}$$

where i is the micro-level entity, $e_{i,t}$ is the forecast error for event $Y_{i,t}$ that can be measured continuously, and $E(Y_{i,t}|I_{t-1})$ is the foreseeable component of $Y_{i,t}$ under information set I_{t-1}. Assuming $e_{i,t}$ follows certain parametric distribution (such as Gaussian in the notation) with time varying mean μ_t and variance $\sigma^2_{i,t}$, then $\sigma^2_{i,t}$ captures the volatility of the unpredictable factor and thus measures uncertainty. Removing the predictable component $E(Y_{i,t}|I_{t-1})$ is essential for getting a clean uncertainty measure. Jurado, Ludvigson, and Ng (2015) use model-based prediction as a proxy. Alternatively, Sheng and Thevenot (2012) use real professional forecasts. The latter method is used in this paper due to the challenge of designing individual prediction models for a large number of firms. In addition, real forecasts from analysts are consistent estimates of true values and are robust to structural breaks.

Since $\sigma^2_{i,t}$ is unobserved, econometric models are needed for its estimation. Traditional models such as Generalized Autoregressive Conditional Heteroskedasticity (GARCH) has been experimented for similar tasks. I choose newer Stochastic Volatility (SV) model for a more desirable results from stochastic paths. I focus on firm-earning as the key variable that generates important firm-level consequences when it is uncertain. Also, an automated data selecting application is designed to extract high-quality firm level uncertainty values from the Institutional Brokers' Estimate System (I/B/E/S) database.

The firm-level uncertainty measure leads to a series of new research directions. First, it breaks down macro uncertainty effects at the firm level and allows studies on macro drivers of uncertainties associated with heterogeneous firms. By matching individual firm's uncertainty dynamics with macro

[4] See Jaynes (1957) for a detailed discussion on subjective and objective probability theories. Similarly, there are two types of uncertainty: subjective uncertainty which reflects a human emotion, or objective uncertainty which concerns the stochastic factor of a data generating process (DGP)

volatility series in the U.S., I find that GDP and financial volatilities are the two major contributors to firm uncertainties with direct effects. The effect of policy uncertainty is also significant, yet shows ambiguous directions. Exchange rate volatility seems to have much stronger impacts on larger firms. Oil price volatility, on the other hand, leads to a countercyclical pattern of uncertainty dynamics for many firms. All 5 macro volatilities combined explain 44% firm-uncertainty variations. A similar study can also be seen in Barrero, Bloom, and Wright (2017). In contrast to their work, I focus on a larger set of macro drivers and adopt a novel empirical approach.

Second, by studying the size distribution of firms in my I/B/E/S sample and the scale effect embedded in Earning Per Share (EPS) values, I derive a unique weights to properly aggregate firm level measurement into macro indices at industrial levels and the country level. The real economic impact of macro uncertainty is tested in a 7 variable VAR framework and results are align with mainstream literature (See Bloom (2009)). More importantly, Gabaix (2011) points out that first-moment shocks associated with large companies generate non-trivial effects on the entire economy. To test such a effect for second moment shocks, I create two uncertainty series associated with the largest 100 U.S. companies and smaller firms respectively. I discover that uncertainties originated from large firms generate similar economic impacts seen for economy-wide shocks. Smaller firms, despite their non-trivial shares in the U.S. economy, lead to insignificant results.

Third, literature has been focusing on macro uncertainty impacts, but to some extent overlooking micro effects following idiosyncratic shocks. Campbell, Lettau, Malkiel, and Xu (2001), Bachmann and Bayer (2013) and Bijapur (2015) are some of the few papers that addresses both. I include both macro and idiosyncratic components of uncertainty in a panel study and regression results show that while both types of uncertainty have negative impacts on firm investments, the impact of macro uncertainty is stronger. In contrast, idiosyncratic uncertainty shows a composite effect: 1. it changes firms' term structure of investment from short-term to long-term and such a change is largely linked to increased spendings in R&D; 2. the direction of its impacts depends on firms' profitabilities: firms with high excess returns benefit from increasing idiosyncratic uncertainty while firms with low returns suffer; Third, its average effect on investment shows a convex path: the negative effect diminishes and possibly turns positive as idiosyncratic uncertainty continuously rise. In addition to above empirical finding, I introduce a composite uncertainty model to interpret the economic sense behind those results.

This paper has several contributions to the growing uncertainty literature. It proposes a novel and reliable firm-level uncertainty measure. It proposes a new micro-founded macro uncertainty index that tracks the U.S. market. It shows the granular uncertainty effects. It bridges the gap between macro and micro empirical studies on the economic uncertainty impacts. It unveils a composite effect of firm-specific uncertainty.

The paper is arranged as follows: Section 2 discusses the firm-level uncertainty measure, its properties, and its macro-drivers; Section 3 proposes a micro-founded macro uncertainty index and the granular origin of its impacts; Section 4 shows the uncertainty decomposition and firm-level uncertainty effects; Section 5 introduces a composite uncertainty model; Section 6 concludes.

2 Measuring Firm Level Uncertainty

2.1 Data

I/B/E/S, the Institutional Brokers' Estimate System, is a database created and maintained by Thomson Reuters. It is a historical earnings estimate database containing analysts' estimates for more than 20 forecast measures. For each company, analysts are asked to make forecasts for as close as current quarter and as far as 10 fiscal years ahead. The one-quarter ahead forecast is the horizon concerned in this paper and corresponding uncertainty measurements are therefore short-term. [5] Figure 1 illustrates the timeline of forecast activities. The I/B/E/S detail dataset records forecast information for 60,000 companies worldwide. In this research, I focus on U.S. firms.

The variable of my main interest is the firms' Earning Per Share (EPS). Despite that variables such as Return on Assets (ROA) or Return on Equity (ROE) allow more straightforward cross-sectional comparisons, EPS has the single longest forecast history and the richest cross-sectional observations essential for an extensive empirical study. [6] The EPS 1 quarter nowcast dataset contains more than 3 million EPS estimates provided by 18,992 financial analysts for 16,724 U.S. firms. Limitations associated with this dataset are discussed in the following sections.

[5] The term structure of firm uncertainty is studied in a separate paper.

[6] While other variable estimates start after 2000, EPS estimates go back to 1982. The Firm Level Uncertainty series extracted form EPS estimates is, therefore, much longer than competing measures in the literature.

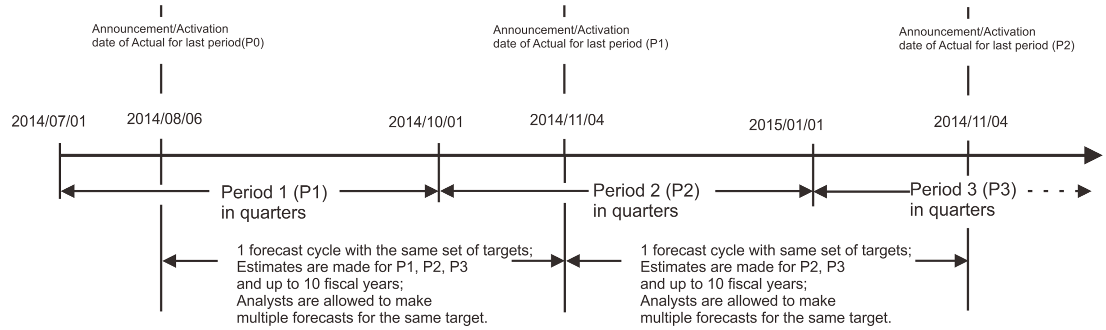


Figure 1: The Timeline of EPS Estimates and Actuals Announcements

Other firm-level variables used in this study are acquired from the Compustat database. Sources for macroeconomic variables and financial series are Federal Reserve Bank of St. Louis and Yahoo Finance.

2.2 Model and Estimation

The time-varying volatility of forecast errors $\sigma_{i,t}^2$ is unobserved, so I rely on econometrics model for its estimation. GARCH is frequently used for such a task. However, GARCH family models assume a serial correlation between the first and second moment and a deterministic path for the second-moment dynamics, both of which make the estimation results strongly correlated with first-moment shocks. Stochastic volatility models, on the other hand, are free of both assumptions and thus return second-moment dynamics beyond first-moment movements. Although the advantage of SV model over GARCH has been discussed in many papers (see Jacquier, Polson, and Rossi (2002), Kim, Shephard, and Chib (1998) etc.), the computational difficulty makes it underused. (See Kastner, Frühwirth-Schnatter, and Lopes (2014)).

The forecast error associated with certain economic target at time t with horizon h is defined as

$$e_{h,t} = Y_{t+h} - E(Y_{t+h}|I_t), \tag{2}$$

where $E(Y_{t+h}|I_t)$ is the conditional forecasts on current information I_t. In order to obtain a quality unpredictable component $e_{h,t}$, it is essential to make conditional forecast $E(Y_{t+h}|I_t)$ as good as possible. Assuming the forecast error is Gaussian $e_{h,t} \sim \mathcal{N}(\mu_{h,t},\, \sigma^2_{h,t})$,[7] and the log-variance process $\sigma_{h,t}$ has an $AR(1)$ autoregressive behavior:

$$\log \sigma^2_{h,t} = \mu_h + \phi_h(\log \sigma^2_{h,t-1} - \mu_h) + \eta_h \epsilon, \tag{3}$$

where ϵ is white noise. Also assuming the initial value $\log \sigma^2_{h,0}$ is drawn from $\mu_h + \frac{\eta_h}{\sqrt{1-\phi_h^2}}\epsilon$, the parameter space left for estimating is $\theta = (\mu_h, \phi_h, \eta_h)$. For each $\mathbf{Y}_h = (Y_{1+h}, Y_{2+h}, ..., Y_{t+h})$, the values of interest $(\sigma^2_{h,1}, \sigma^2_{h,2}, ...\sigma^2_{h,t})$ are recursively and stochastically determined by θ via Metropolis-Hasting sampling within Markov Chain Monte Carlo (MCMC).

2.3 Properties of Firm Level Uncertainty (FLU)

The estimation is conducted on one quarter ahead EPS estimates at quarterly frequency. For the best prediction Y_{it}), the following efforts are made: (1) only the last estimate (best information) from each analyst for each target are kept; (2) only targets that have forecasts from a minimum of 3 different analysts are used; (3) the mean forecast is used to proxy the best estimate in the market. The corresponding forecast errors are calculated together with the actual EPS values from I/B/E/S vintage; (4) only firms that have at least 40 continuous estimates which also satisfy criterion (2) are kept for the model fitting; (5) If a forecast series contains several segments of 40 or more continuous estimates with gaps in between, segments are fitted separately to reflect structure breaks.[8] For my I/B/E/S sample, a total of $1,916$ U.S. firms survive this selection process.

In addition, there are three concerns that require extra attention. First, the forecast error series cannot be autocorrelated. Otherwise, part of these errors is predictable from past information. I use the $AR(1)$ model to detect autocorrelation in each forecast error series. About 45% of the forecast error series show significant autocorrelation at 10% level. For those series, stochastic volatility model fits on the corresponding $AR(1)$ residuals instead of the original series.

Second, correlations among forecast error series lead to aggregation problem in section 3. To

[7]The Gaussian distribution for the forecast error is based on the Central Limit Theorem. As the forecast error is considered to be the composite result of a large number of minor influences associated with both economy phenomena and human activities, its distribution tends to approach Gaussian.

[8]Case studies suggest that most of the forecast breakpoints are caused by shocking events that likely causes structural breaks of a company's volatility series. Those shocking events include merging, company crisis or initiating bankruptcy.

mitigate this concern, I demean each firm-level forecast error series and use excess errors for model fitting. [9] This method is not perfect as excess forecast errors are not orthogonal to $\mu_{m,t}$. However, this drawback has limited effects and likely disappears after aggregation (See Campbell, Lettau, Malkiel, and Xu (2001)). In fact, the consensus forecast errors μ_t has been studied as a macro uncertainty proxy in the literature (See Ozturk and Sheng (2017)). I put little emphasis on this series due to the assumption that macro uncertainty originates from stochastic shocks among a large number of firms and no macro shock can dominate. [10] By the measure construction, macro uncertainty proposed in this paper is different from those derived from market average first-moment shocks. Details regarding my method is discussed in the next sections.

Finally, as I/B/E/S dataset covers a relatively long period, EPS values might change dramatically following large changes to stock prices. The dataset has been adjusted for stock split, but inflation and business growth could lead to a non-trivial scale effect on forecast errors. In other words, large forecast errors could be attributed to larger EPS values but not higher uncertainty. The distribution of all actual EPS series and the distribution of all EPS estimate have thin tails with 99% of values within the range [-4,10]. I remove observations within 1 percentile on both sides to remove extreme values. In addition, I run a correlation test between forecast errors and EPS values which returns a low value 0.004. A similar bivariate regression shows a negligible R^2 value. Both test results imply a trivial scale effect in my measure.[11]

The final fitting results return an unbalanced panel that contains uncertainty measurements for 1,916 U.S. firms. For the rest of the paper, I call them Firm Level Uncertainty (or FLU, or $\sigma_{i,t}$). Figure 2 shows FLU for some well-known U.S. companies. The uncertainty associated with Apple is very sensitive to new product announcements and former CEO Steve Jobs' death. Amazon's uncertainty rises substantially in recent years due to its fast expanding. Starbucks, on the other hand, experiences high uncertainty when it was forced to close more than 300 stores during the 07-09

[9]Specifically, I use the market average forecast error $\mu_{m,t}$ as the distribution mean in equation (1), or $e_{i,t} = \mu_{m,t} + \sigma_{i,t}\epsilon$. $\mu_{m,t}$ is obtained by collapsing individual forecast errors sequentially along the dimension of analysts with pegged firm i and time t then along the dimension of firms with pegged t.

[10]This is confirmed in section 2.4 as no macro shock have universal impacts on all firms.

[11]For a more strict robustness check, I run the following regression

$$\sigma_{i,t}^2 = a_i + EPS_{i,t} + \xi_{i,t} \tag{4}$$

at the firm level and use $a_{i,t} + \xi_{i,t}$ as the scale-free uncertainty measure. This measure returns very similar results at both macro and micro level. Graphs based on $\xi_{i,t}$ might be provided upon request.

recession. In addition, its uncertainty is closely linked to its oversea business. Bank of America' uncertainty is extremely sensitive to financial crises and recessions; Major mergers and acquisitions also trigger large uncertainty spikes.

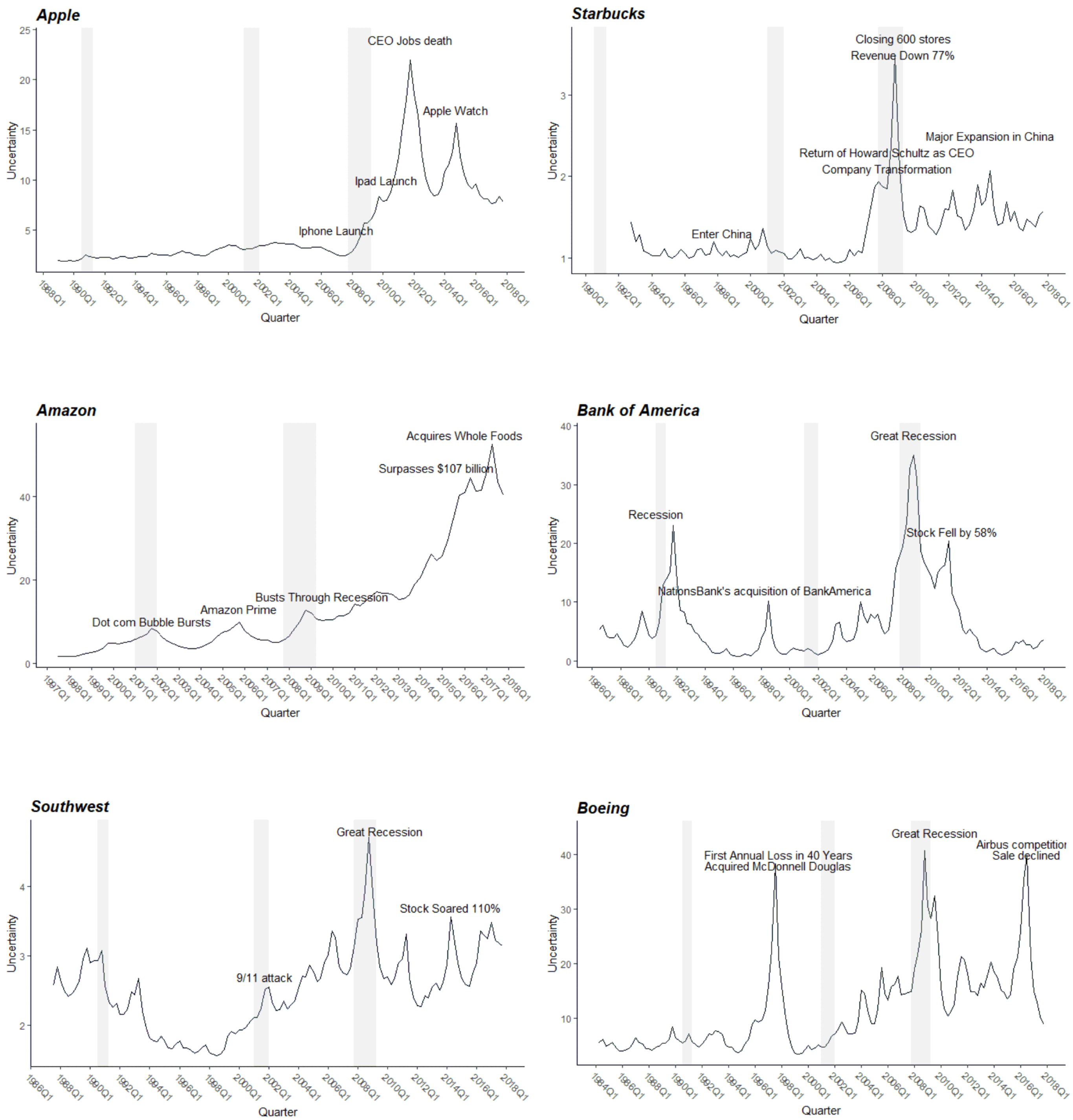


Figure 2: Uncertainty indices of Well-known U.S. Companies (FLU)

2.4 What Drives Firm Level Uncertainty?

Analysts observe the economic consequences associated with firm uncertainties, but they might not see the cause. In this section, I focus on macro drivers of firm uncertainties by examining the link between FLU and 5 major macroeconomic volatilities. The remaining firm-specific (idiosyncratic) uncertainty effects are covered in section 4.

The set of U.S. macro variables includes Real GDP, Oil price, Exchange rate, Economic policy, and Stock price.[12] [13] These macro variables cover different aspects of the U.S. economy and are believed to have universal impact on most firms. Data for GDP, oil price and exchange rate are first-moments so I apply stochastic volatility model to their detrended log-series to derive their implied volatilities. I choose not to use realized volatility due to the strong backward-looking nature of this method and the low frequency of macro data.[14] EPU and VIX, on the other hand, are natural second-moment measures and are used without treatments. Figure 3 illustrates the volatilities of these 5 macro series. They respond similarly to large economic and political shocks but display distinct persistence, timing, and magnitude upon small or medium shocks.

[12] The exchange rate is a weighted average of the foreign exchange value of the U.S. dollar against the currencies of a broad group of major U.S. trading partners. The stock price refers to S&P 500 index and its volatility is proxied by VIX. The economic policy refers to EPU proposed by Baker, Bloom, and Davis (2016).

[13] Data for real GDP, Exchange rate are acquired from St. Louis FRED. VIX data is acquired from the CBOE website. EPU is downloaded from the Economic Policy Uncertainty Index website.

[14] Using realized volatilities of these macro variables returns lagged peak points corresponding to shocking events.

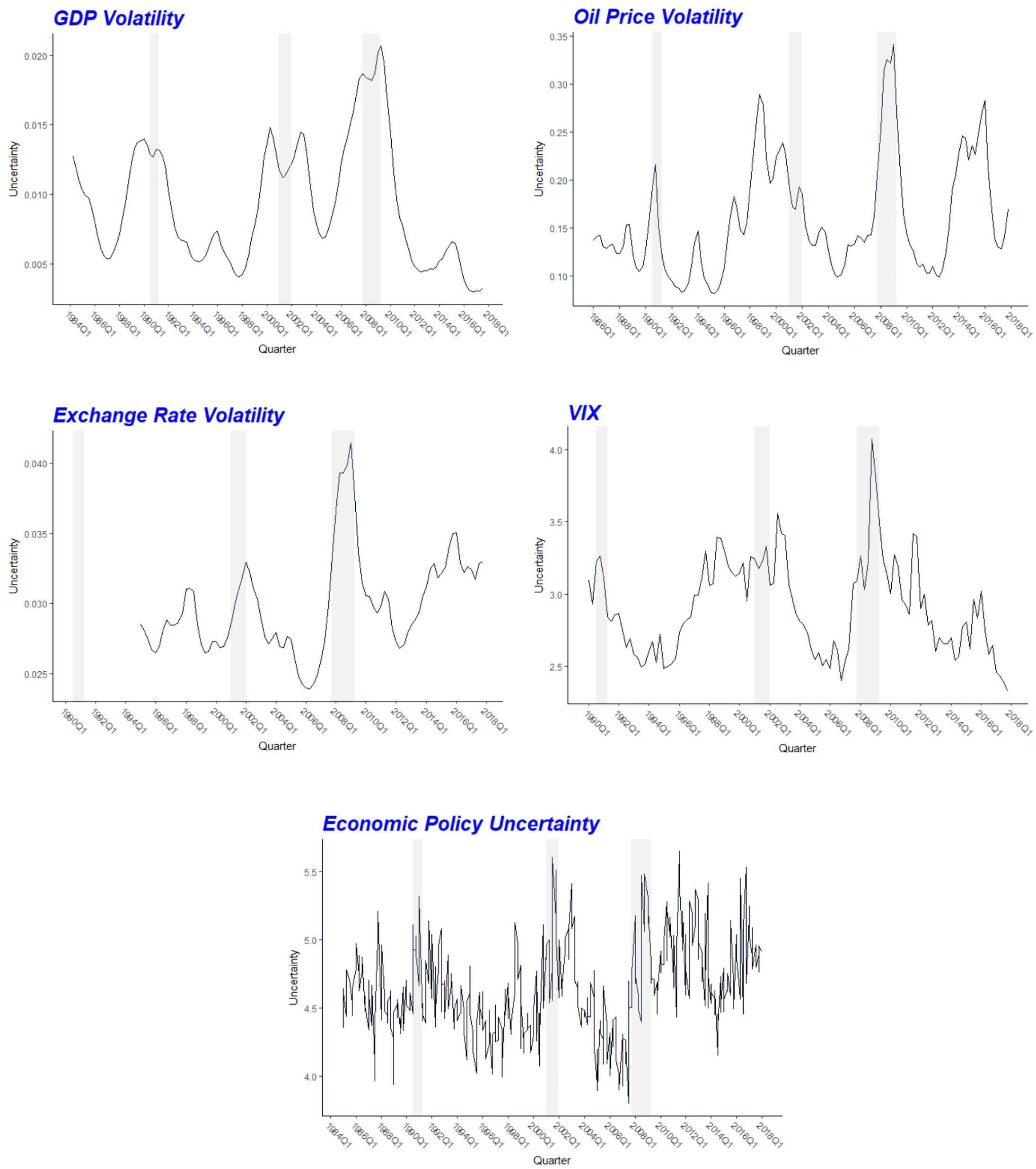


Figure 3: Volatilities of Macro Variables

Volatilities for GDP, Oil price and Exchange rate are computed by fitting stochastic volatility models on their detrended log-series. EPU and VIX are log-series. Data for real GDP, Exchange rate are acquired from St. Louis FRED. VIX data is acquired from the CBOE website. EPU is downloaded from the Economic Policy Uncertainty Index website.

Literature has extensive studies on macro impacts of those volatility series, but their firm-level breakdown is largely overlooked. To see how firms respond to those macro uncertainty shocks individually, I use regressions:

$$\sigma_{i,t}^2 = c_i + \boldsymbol{\beta}_i \boldsymbol{X}_t + \zeta_i \quad \text{for each i,} \tag{5}$$

where $\sigma_{i,t}^2$ is FLU; $\boldsymbol{X}_t$ is the vector of macroeconomic volatility series including GDP volatility, Oil price volatility, Exchange rate volatility, Economic policy uncertainty, and VIX; ζ_i is disturbance with firm-specific variance. Since most of the companies in my sample are not large enough to generate visible impacts on the entire economy but vulnerable to fluctuations in the market, those regression results provide valuable information on the causes of FLU.

In Table 1 upper panel, I report three statistics obtained from $\boldsymbol{\beta}_i$ and p_i values from $1,865$ individual regressions:[15] (1) Response rate: the share of firms that respond to certain macro volatilities at 10% significant level ($p_i \leq 0.1$); (2) Positive: the share of firms that have significant and positive response ($p_i \leq 0.1$ and $\beta_i \geq 0$); (3) Negative: the share of significant but negative responses ($p_i \leq 0.1$ and $\beta_i < 0$). Among 5 macro volatilities, policy uncertainty, GDP volatility and oil volatility have slightly higher response rates that implies more universal impacts. In addition, firm uncertainties present both pro-cyclical and counter-cyclical patterns depending on the type of drivers. Stock market volatility shows a slightly lower response rate, but has a large positive share for a direct impact of the financial market uncertainty on firms' performance. Meanwhile, firms' responses to oil price uncertainty show a counter-cyclical pattern, which is not surprising considering that unstable oil price would encourage more domestic oil production which not only benefits many domestic companies but stabilizes oil price expectation. The effect of policy uncertainty is a mixed bag: while some firms' uncertainties rise after an unspecified policy uncertainty shocks some other firms fall. The nearly equal split in the data implies policy biases. GDP volatility displays the most dominating negative effect on firms' earning stability. The exchange rate volatility turns out to be less influential in this micro setup. However, as we see in the next section, its macro-level effects are much stronger.

In the lower panel, I report the mean R^2 of all $1,865$ regressions and the percentage change of mean R^2 after excluding 1 of 5 macro volatility. On average, nearly half of FLU variations might be explained by the aggregate effect of these 5 macro volatilities. In addition, GDP and stock market volatilities contribute most to firms' uncertainty fluctuations.

[15] The sample size shrinks slightly as a result of merging with other databases.

Statistics	Politics	Stock Market	GDP	Oil Price	Exchange rate
Response Rate	0.55	0.37	0.51	0.51	0.37
Positive	0.28	0.31	0.43	0.12	0.23
Negative	0.26	0.06	0.07	0.38	0.14
Mean R^2			0.44		
Change of R^2	0.09	0.14	0.18	0.09	0.13

Note: Results are based on β and p values from 1,865 individual regressions. Numbers show the share of firms have p values smaller than 0.1, the share of positive β and negative β conditional on $p \leq 0.1$.

Note: Mean R^2 is the mean of $1,865$ regression results. Change of R^2 is the percentage change of mean R^2 after removing one of 5 variables in regressions.

Table 1: Macro Drivers of Firm-Level Uncertainty

3 Constructing Macro Uncertainty Index

3.1 Estimating the Weighting Scheme

The reason for constructing a micro-founded macro uncertainty measure is threefold: (1) Policymakers are more interested in macro uncertainty since their policy goals are usually at the macro scale; (2) Existing macro uncertainty measures are usually built on macro variables such as GDP, inflation, stock market indexes, which contains little micro-level information; (3) An effective measure of idiosyncratic uncertainty requires removing the macro (or common) component. A further discussion of of this topic is provided in section 4.

Two challenges are unavoidable while aggregating the microdata: 1. the panel is unbalanced and the weights need to be specific. Liu and Sheng (2018) study the panel composition issues in the SPF dataset and show that changes of panel composition substantially drive estimation results. Unfortunately, both firms and forecasters in I/B/E/S database varies from time to time. To mitigate such issues, I limit the uncertainty episodes to periods that have enough overlapping firms as well as sufficient participating forecasters. The remaining sample has an average of 50% shared companies.

2. A specific weighting scheme for FLU is required due to the nature of EPS. Standard methods such as mean or median might not be appropriate for aggregation due to significant size-effect (see Gabaix (2011)).

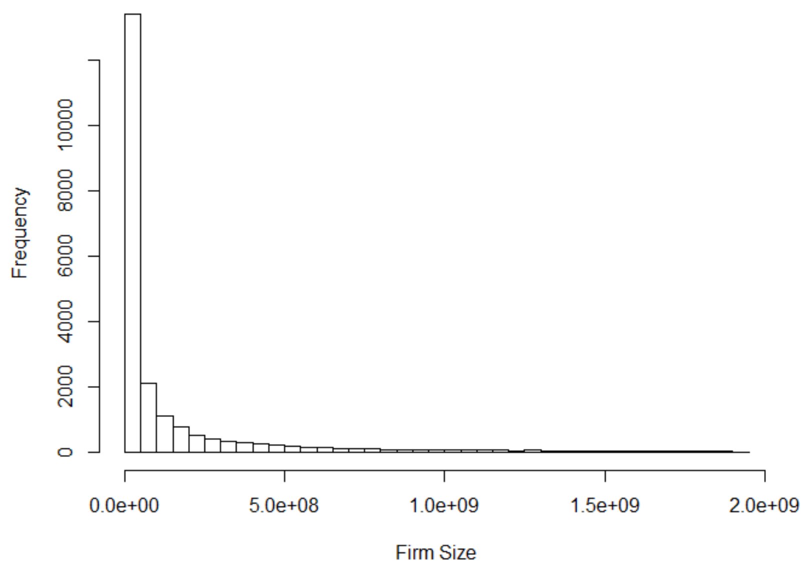


Figure 4: The Power-Law Distribution of I/B/E/S Firm Size

The firm size is proxied by market capitalization. Using company earnings renders similar results.

The size distribution of firms in the I/B/E/S sample follows a power-law as shown in Figure 4.[16] This result is consistent with Axtell (2001) who uses the Census data to show a Zipf law distribution of firm size. With a fat-tailed distribution, mean or median are inappropriate statistic for aggregation. In addition, EPS value contains built-in scale as the number of outstanding shares that requires proper controls as well. The weight calculation starts with a macro target: supposing that the process of unpredictable part of EPS movements within arbitrary short time interval t follows Brownian motion with μ_t and $\sigma_{i,t}$, which are firm and time specific. [17] The stochastic volatility model returns the latent volatility $\sigma_{i,t}$ associated with firm i's unpredictable EPS. However, policymakers' focus is the uncertainty of the whole economy, so they are interested in the volatility of unpredictable

[16] The size is proxied by firms' market capitalization. For robustness check, I also use company total earnings as size for the distribution plot and the results are similar.

[17] Assuming Brownian motion for EPS process omits both predictable dynamics in each EPS forecast error series as well as correlations among different series. In practice, adopting such an assumption requires that raw data to be treated with AR(1) test and demeaning as discussed above.

total earning M_t, which is the sum of all individual firm earnings. The variance associated with the total earning growth $\frac{\Delta M_t}{|M_t|}$ is broken down into the weighted average of individual $\sigma_{i,t}$ with weights determined by $w_{i,t} = (\frac{n_{i,t}}{|\sum_{\mathbf{i}} s_{i,t}n_{i,t}|})^2$, where $s_{i,t}$ is EPS and $n_{i,t}$ is the outstanding shares traded in the market. The derivation is shown as follows:

$$\Delta s_i = \sigma_{i,t}\epsilon_i \tag{6}$$

$$M_t = \sum_{\mathbf{i}} s_{i,t}n_{i,t} \tag{7}$$

$$\frac{\Delta M_t}{|M_t|} = \frac{1}{|M_t|}\sum_{\mathbf{i}} \Delta s_{i,t}n_{i,t} = \sum_{\mathbf{i}} \sigma_{i,t}\frac{n_{i,t}}{|\sum_{\mathbf{i}} s_{i,t}n_{i,t}|}\epsilon_i \tag{8}$$

Taking variance on both sides we get

$$\sigma^2_{M,t} = \sum_{\mathbf{i}} \sigma^2_{i,t}\Big(\frac{n_{i,t}}{|\sum_{\mathbf{i}} s_{i,t}n_{i,t}|}\Big)^2 \tag{9}$$

The result implies that in an extreme case where all firms have identical earning and volatility, the market volatility decays according to $\frac{1}{s_t\sqrt{N}}$ where N is the number of firms. As N goes to infinity, market volatility converges to 0 with probability 1. However, a fat-tailed distribution and idiosyncratic volatilities would not give such a converge. Using the weights in equation (1.9), I aggregate FLU at the industrial level, as shown in Figure 5.

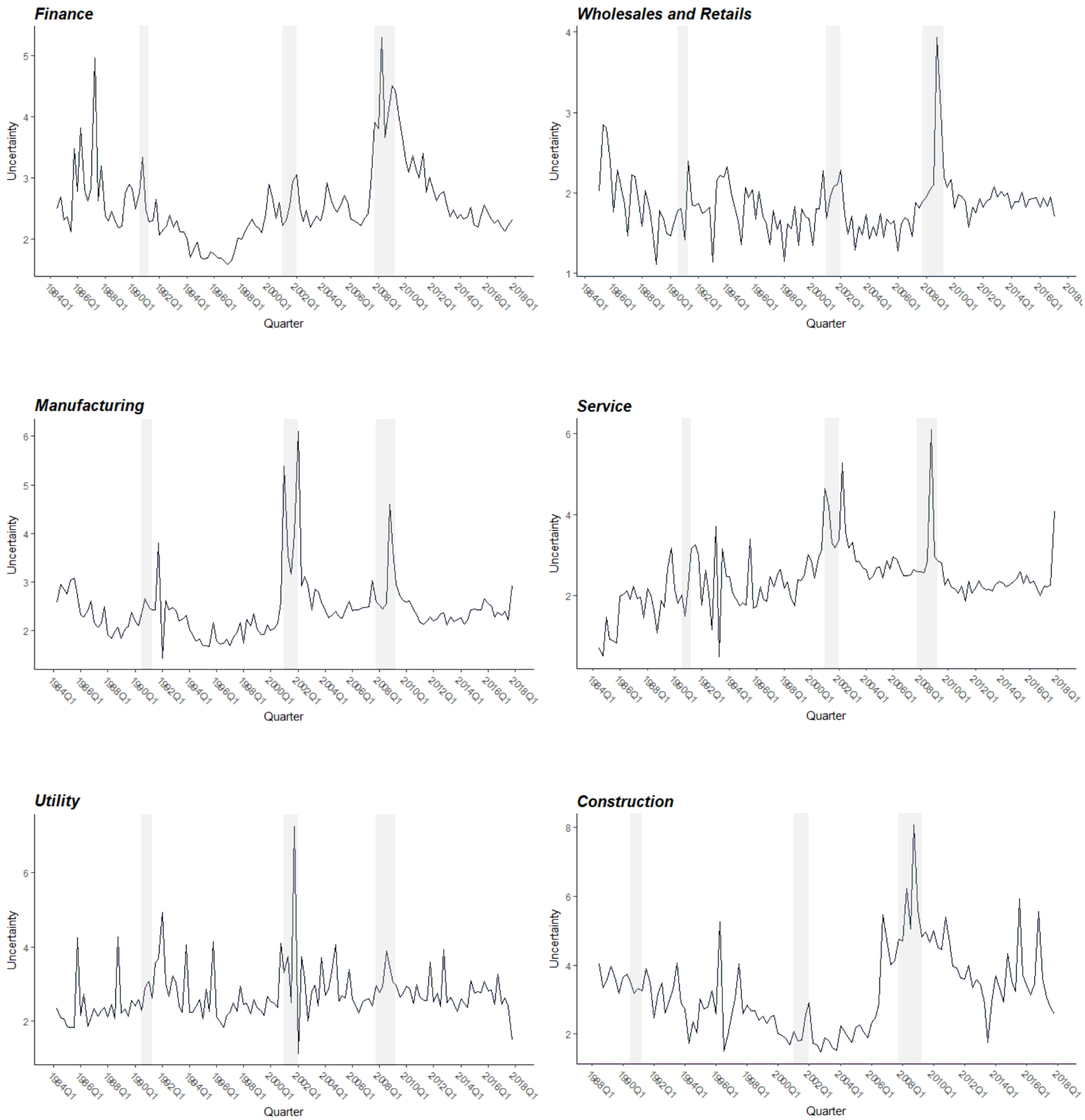


Figure 5: Industrial Level Uncertainties

I focus on two special shocking events to examine uncertainty effects across industries: 2007-09 Great Recession started with the collapse of banking then spread out to the whole economy. Its widespread impacts are seen in four major industries – finance, retails, manufacturing, and construction (housing). The fact that uncertainty spikes in financial sector appears before other industries indicates the origin of 07-09 recession. By comparison, the 9/11 terrorist attack leads to immediate large uncertainty spikes in manufacturing and utility sectors. It is largely attributed to the surge of military related good and energy demand in preparation for the war. The magnitude and timing of those uncertainty spikes imply that different types of uncertainty shocks have uneven impacts among industries and cause different economic consequences.

Figure 6 shows the short-term Macro Uncertainty index (or MU, or $\sigma_{M,t}$) based on the total earning growth associated with all $1,865$ U.S. firms using the weighting scheme produced in equation (9). [18] [19] This index captures the aggregated uncertainty dynamics in FLU, and appears to be sensitive to recessions, financial crises, wars, terrorist attacks, and presidential elections. Three recession episodes post 1984 are marked with large uncertainty spikes. The 2007-09 recession shows largest uncertainty spike. In addition, Gulf War I, 9/11, Iraq War, and recent terrorist attacks all trigger high levels of uncertainty. The great moderation in the 90s shows the lowest uncertainty level of the entire observed period, and the uncertainty level post the moderation (2001Q2 as cutoff) is significantly higher even without large spikes around 2009. [20] In general, MU well tracks all shocking events in recent U.S. history and the magnitude of spikes are largely consistent with events impacts. In the next section, I compare the uncertainty measures in this paper with other measures in the literature.

3.2 Comparisons with Other Uncertainty Measures

3.2.1 Firm-Level Measures

Firm-level uncertainty measures are still uncommon in the literature, but we have alternative measures such as Barrero, Bloom, and Wright (2017) which uses firm-level implied volatilities; Hassan, Hollander, van Lent, and Tahoun (2019) which features qualitative data of earning conference call

[18] The utility sector is excluded from the aggregation due to its noisy pattern. Nevertheless, the full sample returns a similar macro uncertainty dynamics.

[19] As a robustness check, I repeat this experiment with GARCH(1,1) model and the results are qualitatively similar. However, the SV model returns more precise responses to large shocking events.

[20] The elevated uncertainty level post 2001Q2 is confirmed with a one-side t-test at 1% level. Peak points surrounding the 2007-09 recession are excluded.

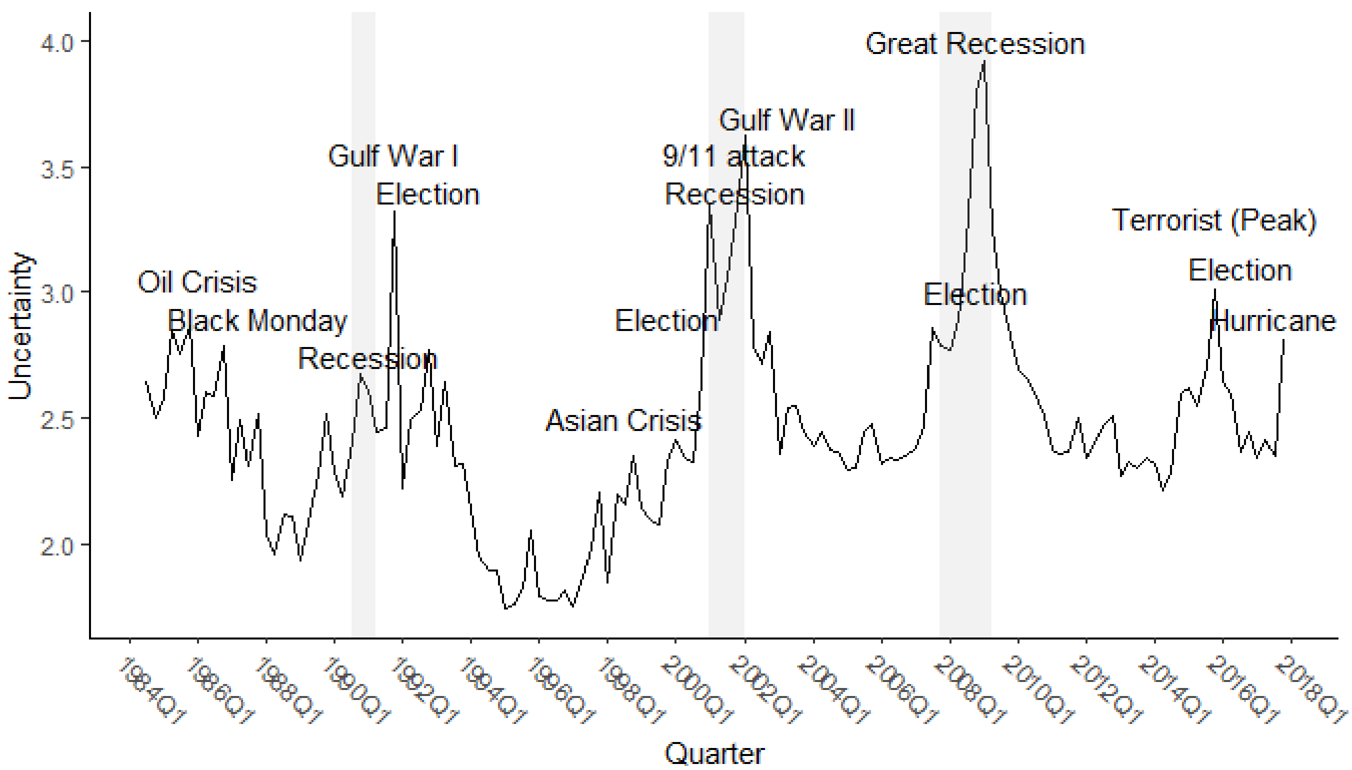


Figure 6: Macro Uncertainty Index (MU)

memos, Campbell, Lettau, Malkiel, and Xu (2001) which uses realized volatility of stock returns. Moreover, old method such as forecaster disagreement can also be applied to firm-level data. In this section, I compare FLU with the qualitative firm uncertainty measure in Hassan, Hollander, van Lent, and Tahoun (2019) and use disagreement as a baseline.

Uncertainty measures based on qualitative studies have drawn increasing attention in recent years. Baker et al. (2016) expand EPU measure to 22 regions and this index is being used extensively in related researches. Hassan, Hollander, van Lent, and Tahoun (2019) extend this methodology with a more sophisticated text scanning algorithm on company quarterly earning conference call memos and build firm risk indices associated with political risks, non-political risks, and general risks. My comparison focuses on their Firm General Risks (FGR) because drivers of FLU are multifaceted. At the firm level, the correlation between FLU and FGR is low around 0.13. However, if both measures are aggregated into macro series, [21] the correlation jumps up to 0.5. [22] To see the reason, I compare two measures side by side with the same set of companies and discover that the low firm-level correlation is largely due to the much higher volatilities in FGR as shown in Figure 7. However, much of those volatilities offset during aggregation and leaves a similar macro dynamics. In general, these two measures are based on largely different methods and captures different types of uncertainty. Both indecies could be a good supplement to the other and researches are encourage to use different measures for a more complete picture.

[21] I use the weighted average of FLU and the cross-sectional mean of FGR following Hassan, Hollander, van Lent, and Tahoun (2019).

[22] The major gap between these two measures are found to be prior to 2003. FGR shows no spike for the 2001 recession and the Iraq war periods while my measure shows large spikes. If both measures are compared post-2003, the correlation at the macro level further increases to 0.71.

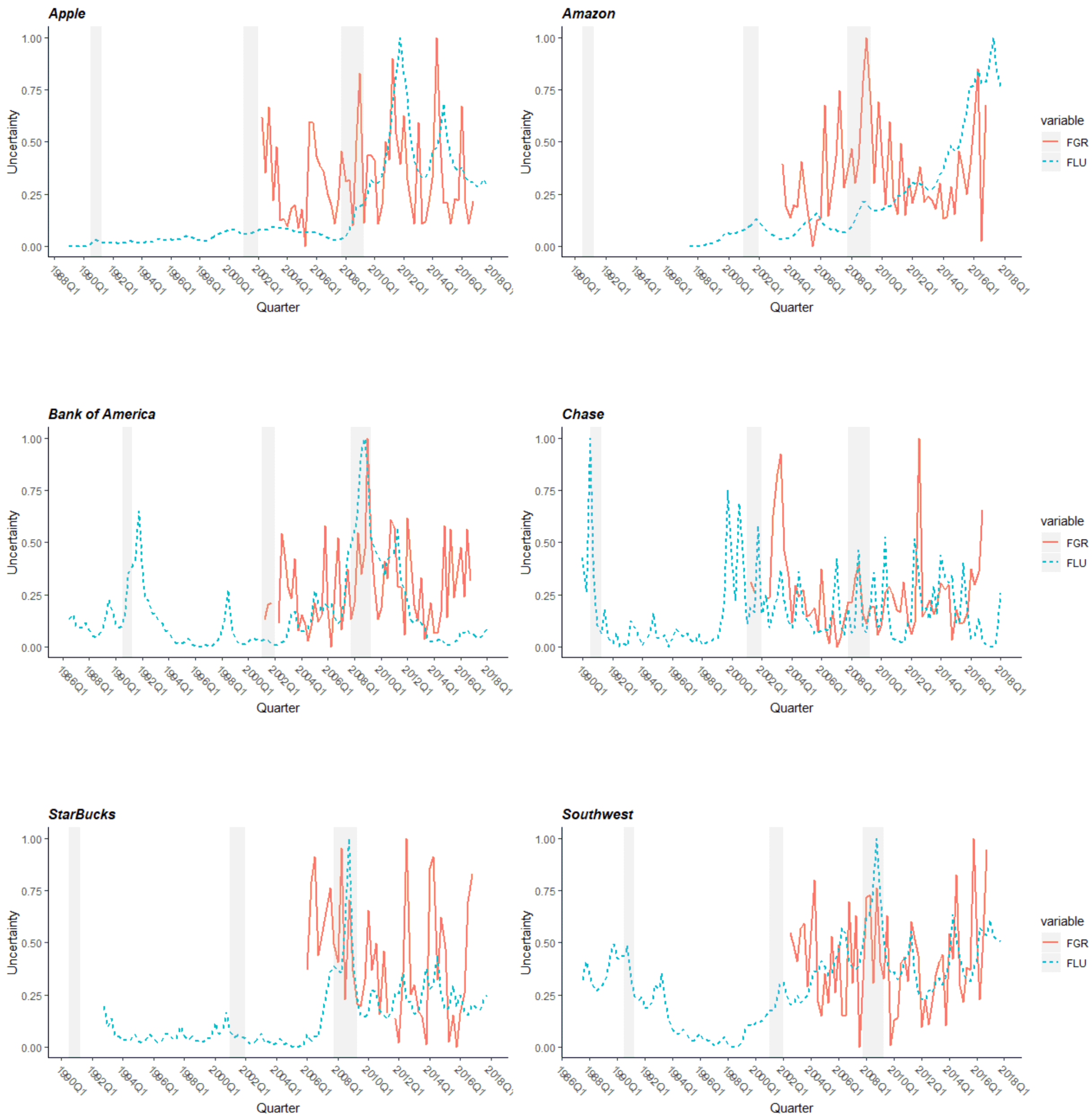


Figure 7: Firm-Level Comparisons with Firm General Risks (FGR)

FGR refers to Firm General Risk proposed by Hassan et al. (2017); FLU refers to my measurements.

The forecast disagreement is defined as the standard deviation of cross-sectional forecasts associated with a target. It is an ex-ante uncertainty measure that can be tracked in real time. Although FLU is an ex-post measure, the link between these two could be tight – a highly unpredictable economic environment would produce noisy signals that leads to diversified opinions on the future. If this hypothesis is valid then we should expect a high correlation between these two measures. At the firm level, the correlation between these two is 0.6. If both measures are compared after aggregation, the correlation rises to 0.9. The high macro-level correlation implies that micro-level noises are largely canceled out during aggregation. The correlation values imply that while FLU adds granularity into existing measures, its macro-level performance is largely supported by old methods.

Figure 8 shows the aggregated results for all three firm-level measurements. The major disagreement can be seen as the unidentified plummets during the 2001 recession in FGR. The main advantage of FLU is its much longer history. [23]

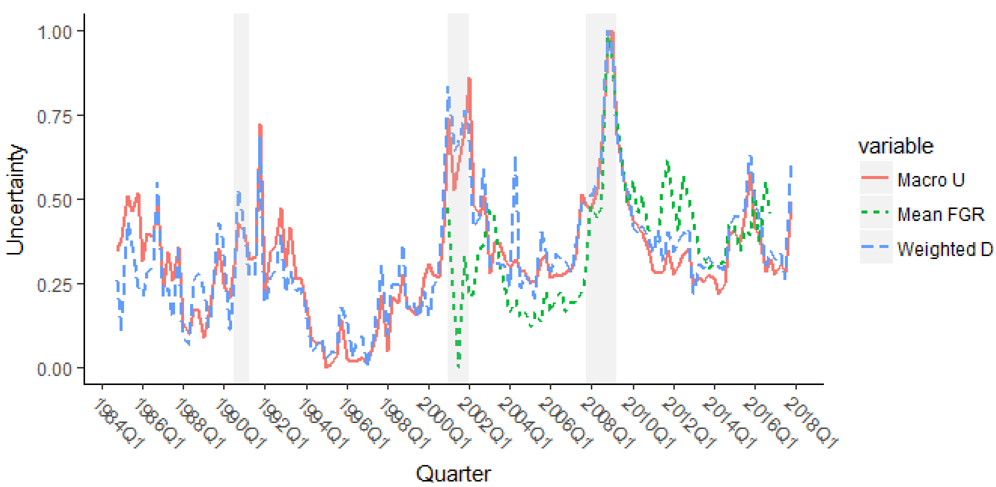


Figure 8: Comparison of Firm Uncertainty Measures

Mean FGR is the cross-sectional mean value of firm general risks in Hassan et al. (2017). Weighted Disagreement uses the same weighting scheme in equation (9).

[23]It is worth noting that FLU captures the 1 quarter ahead short-term uncertainty, but FGR does not have a clear term specification. Barrero et al. (2017) argue that the term structure is a non-trivial factor of uncertainty impacts, which might also explain the low correlation between these two measures at the firm level.

3.2.2 Macro Measures

For economy-wide uncertainty measures comparison, I include three macro uncertainty measures frequently used to monitor the U.S. market: (1) VIX, often referred to as "investor fear gauge", is the implied volatility based on S&P 500 index options; (2) EPU, a well-known economic policy uncertainty index constructed from newspaper (See Baker et al. (2016)); (3) JLN, a macro uncertainty measure based on second-moment co-movements among a large number of macro variables. (see Jurado, Ludvigson, and Ng (2015)). Table 2 shows the correlation between MU index and the above three popular measures.[24] MU is highly correlated with all three measures with the highest correlation with JLN. This is expected as EPU and VIX only capture the political and financial side of U.S. economy, while JLN incorporates many U.S. macro variables and matches MU's scope. The exact movements of all four macro uncertainty indexes are shown in Figure 9.

	MU	EPU	VIX	JLN	GDP Vol	Oil Vol	Exchange Vol
MU	1.00						
EPU	0.52	1.00					
VIX	0.56	0.57	1.00				
JLN	0.82	0.33	0.54	1.00			
GDP Vol	0.66	0.06	0.41	0.78	1.00		
Oil Vol	0.44	0.11	0.49	0.55	0.40	1.00	
Exchange Vol	0.63	0.45	0.54	0.65	0.40	0.66	1.00

Note: The measures include the VIX in Bloom (2009), the EPU in Baker et al.(2016), the JLN index in Jurado et al. (2015), and Macro Uncertainty introduced in this paper.

Table 2: Macro-Uncertainty Correlations

[24] I use the 3 months ahead uncertainty index in Jurado et al. (2015) to match the forecast horizon in my I/B/E/S sample. It is labeled as JLN.

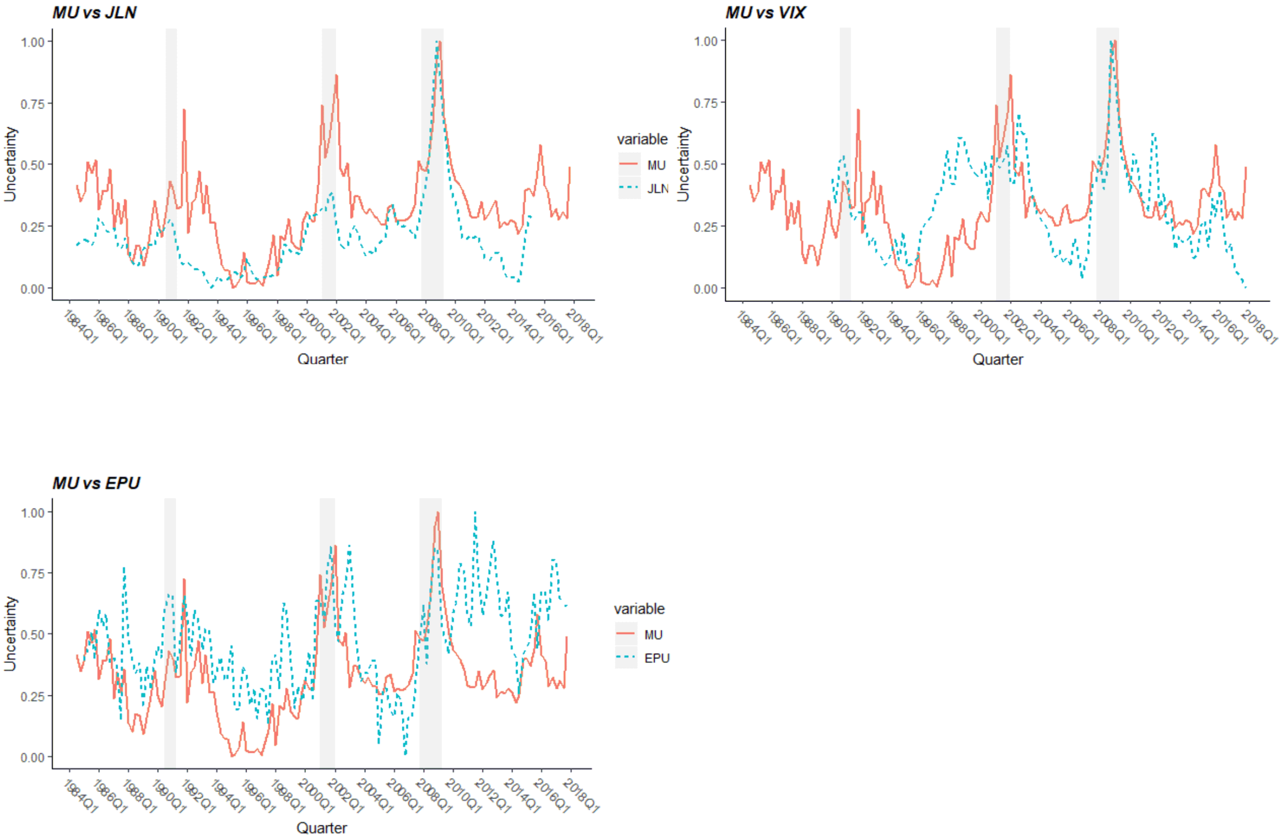


Figure 9: Comparison of Macro Uncertainty Measures

EPU refers to Economic Policy Uncertainty (Baker et al. (2016)); VIX is the uncertainty index provided by CBOE; JLN refers to the macro uncertainty index in Jurado et al. (2015).

In addition, the correlation between MU and volatilities associated with major macro variables are also reported in Table 2. MU captures a good portion of volatilities associated with GDP, oil price, and exchange rate with its highest correlation with GDP and lowest with oil price. It is worth noting that exchange rate is highly correlated with MU despite that it shows a relatively low response rate in section 2.4. The improvement is largely due to the weight used for aggregating micro data. MU puts heavier weights on larger firms which are more vulnerable to unstable exchange rates due to their extensive international businesses. However, the response rate in Table 1 puts equal weights on all firms so the strong effect on large firms are diluted by large number of smaller firms who have less or no international businesses. The low correlation with oil price volatility could be attributed to three facts: 1. Top U.S. companies are largely technology or finance oriented so their businesses are not tightly linked to fossil fuel; 2. Increasing volatility in oil price has diversified impacts on firms across industries and those idiosyncratic effects cancel out during aggregation; 3. Oil price pass through has fallen over time as firms become less fossil fuel dependent in general.

The comparison from both micro and macro prospectives shows advantages of the proposed measure FLU: 1. It tracks a much longer history than competing firm level measurements; 2. It returns less volatile and more explainable results; 3. Its macro-level dynamics after aggregations are largely supported by old methods; 4. It has the potential to expand to more firms and longer horizons.

3.3 The Macro Impact of Uncertainty

Mainstream theories suggest a negative real impact of uncertainty at the macro level and this is largely supported by data. The major disagreement is whether the initial drop of investment and employment is followed by a strong rebound. Bloom (2009) gives theoretical insights on rebounds post uncertainty shocks. However, such a view is challenged by Jurado, Ludvigson, and Ng (2015) and their VAR results show a more persistent drop with no significant rebound. Following Bloom (2009), I include Macro Uncertainty in a 7 variable VAR framework: [25] [26]

$$\begin{bmatrix} \text{log(S\&P 500 Index)} \\ \text{Uncertainty (MU)} \\ \text{log(Wage)} \\ \text{Federal Funds Rate} \\ \text{log(CPI)} \\ \text{Unemployment Rate} \\ \text{log(Industrial Production)} \end{bmatrix}$$

[25] All macro data are downloaded from St.Louis FRED or Yahoo Finance. All variables in level are rescaled with their log-values. Variables shown non-stationary due to time trend are detrended by HP filter. Cholesky decomposition is used to identify shocks through ordering.

[26] The order of these variables is based on their sensitivities and adjustment speeds to shocks. Such an order is in line with the literature.

Impulse response functions for both industrial production and unemployment are shown in figure 10 with 90% confident bands. A one standard deviation shock to macro uncertainty is accompanied by a 0.55% industrial production drop and a 0.12 increase in unemployment rate roughly 6 months post the shock. A significant rebound for industrial production is observed after 1 year, but a similar rebound is not clearly found for unemployment. This result is to large extend consistent with Bloom (2009).

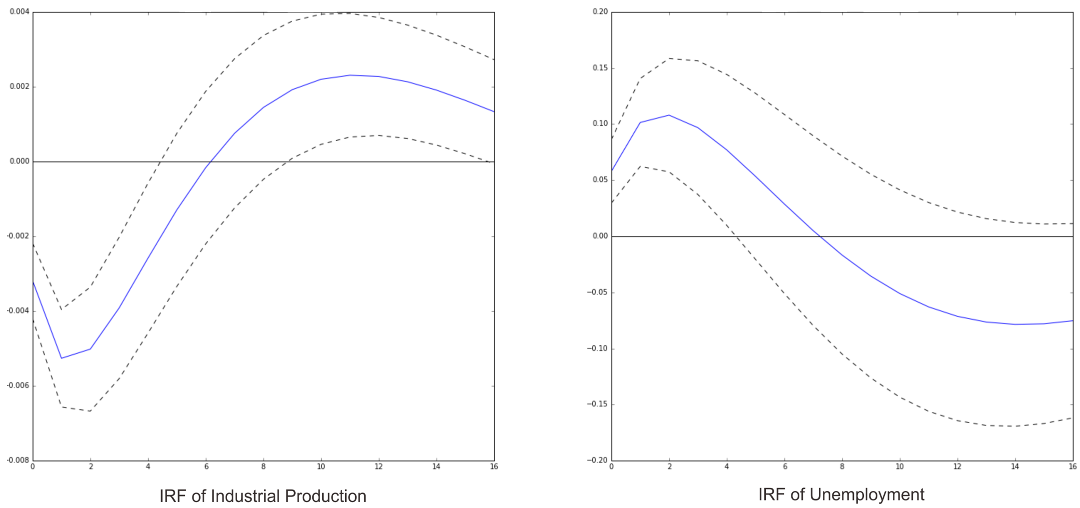


Figure 10: Responses of Industrial Production and Unemployment to Macro Uncertainty (MU) shocks (90% Confidence Interval)

While the macro uncertainty impact has been extensively covered in the literature, micro effects are largely missing in the literature due to the lack of reliable micro-level measurements. The following section takes advantage of the micro-information in FLU and add granularity to existing macro uncertainty effects.

3.4 The Granular Origin of Macro Uncertainty

Using the same weight, I aggregate up two additional uncertainty series – one for the largest 100 U.S. firms and the other for remaining smaller firms. At each t, the average number of firms is 906 and the average market share of top 100 companies is 51.8%. [27] Figure 11 illustrates these two series alongside MU. Not only the uncertainty associated with smaller firms is higher than that of large firms, but the volatility of uncertainty is also higher. The former result is consistent with Stanley,

[27] The market share is calculated based on company earnings.

Amaral, Buldyrev, Havlin, Leschhorn, Maass, Salinger, and Stanley (1996), while the latter implies a negative relationship between firm size and the volatility of firm uncertainty. [28] To my understanding this is the first time such a relationship is documented in the data.

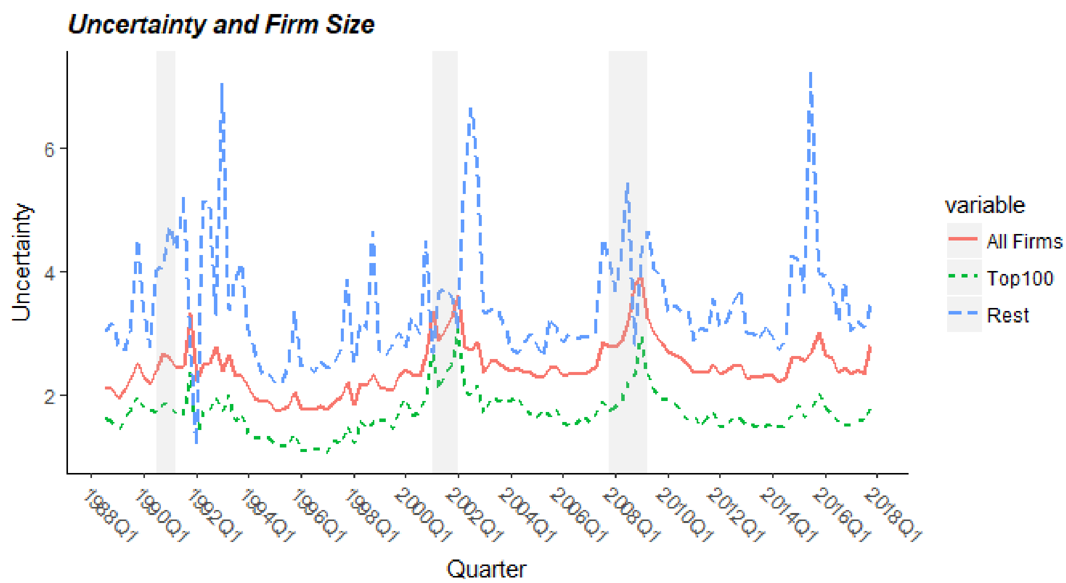


Figure 11: Uncertainty and Firm Size

To test the granular origin of macro uncertainty impact, these two uncertainty series are included in the above VAR framework. Impulse response functions are reported in Figure 12 and Figure 13. The IRFs for top 100 firms are very similar to those from the whole sample with slightly wider confident bands. By comparison, the IRFs for smaller firms show much smaller impacts. The pairwise difference between Figure 12 and Figure 13 is illustrated in Figure 14 with 10% confident band. The overall differences between two set of IRF regression results are statistically significant at 1% level. Regression and testing results together confirm that despite smaller firms having a non-trivial share of U.S. economy and also a overall higher uncertainty level, real macroeconomic impacts are mainly generated by uncertainty shocks associated with large companies. This conclusion is

[28]The higher volatility of uncertainties for small firms are confirmed with one-side t test.

consistent with Gabaix (2011). However, Gabaix (2011) only focus on the granular effects associated with first-moment shocks which is not strictly uncertainty. In this paper, all shocks are based on second-moment dynamics and similar conclusion is reached.

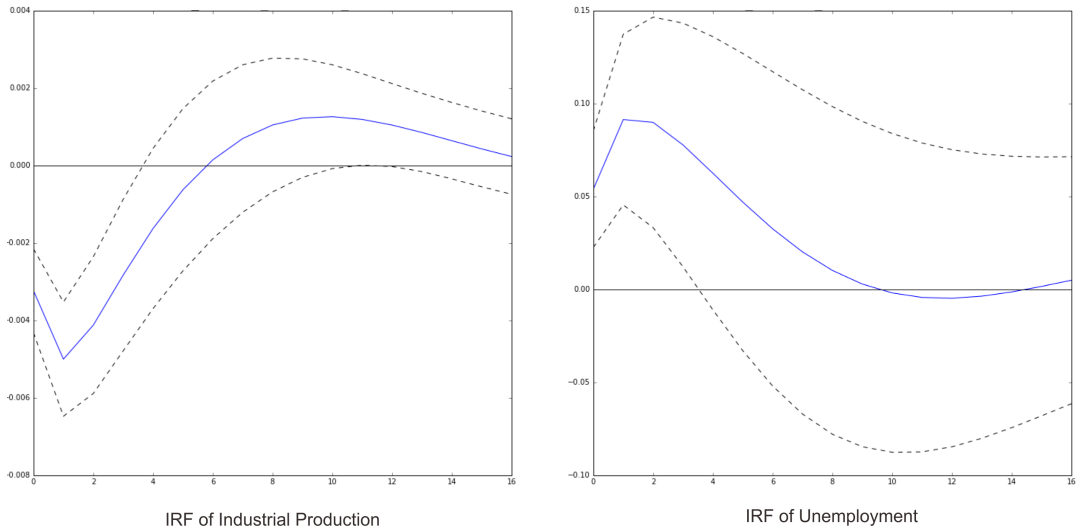


Figure 12: Responses of Industrial Production and Unemployment to 100 Largest U.S. Firms' Common Uncertainty Shocks (90% Confidence Interval)

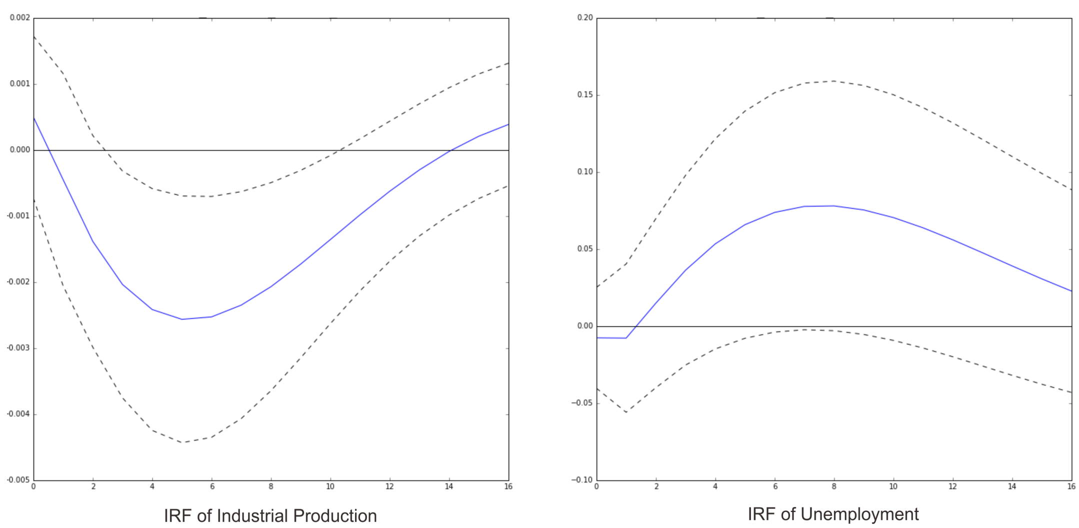


Figure 13: Responses of Industrial Production and Unemployment to Shocks on the Common Uncertainty Excluding TOP 100 (90% Confidence Interval)

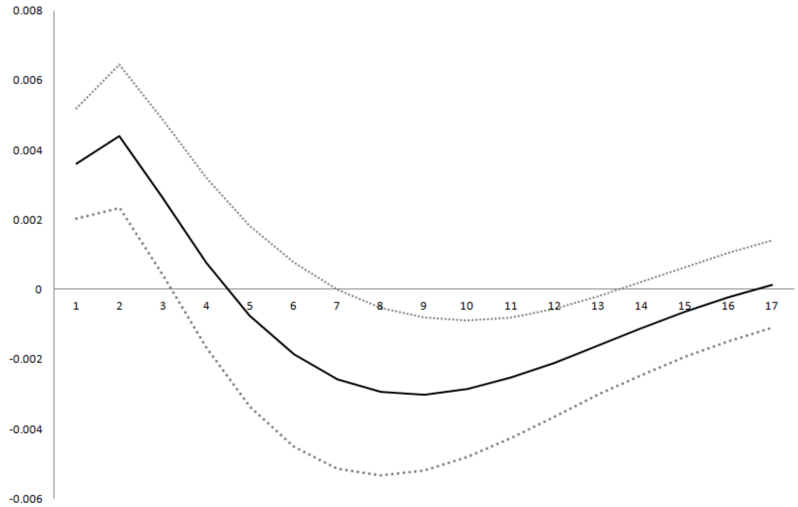


Figure 14: Difference in IRFs between Top 100 Firms and the Rest (90% Confidence Interval)

4 The Impact of Firm-Specific Uncertainty

4.1 Decomposing the Firm Level Uncertainty

In section 2.4, I show that FLU contains non-trivial share of macro volatilities not specific to individual firms. As a result, using FLU directly in a micro regression setup would not separate out the effect of "true" idiosyncratic uncertainty from the macro ones. We might overlook important dynamics exclusive to each component by only looking at their composite value alone. In addition, while macro uncertainty is usually exogenous to smaller producers, idiosyncratic uncertainties are tied to day-to-day business and thus more important to company managers. To distinguish uncertainty effects on different scales, FLU is decomposed into macro and firm-specific series.

Similar decomposition based on Capital Assets Pricing Model(CAPM) was seen in Campbell, Lettau, Malkiel, and Xu (2001). Their decomposition starts with regressions on the first moment shocks,

$$e_{i,t} = \widetilde{\beta}_{i,c} e_{c,t} + \eta_{i,t} \tag{10}$$

where $e_{i,t}$ is the idiosyncratic first-moment shocks, $e_{i,c}$ is market average first-moment shock and $\eta_{i,t}$ is the regression residual. Taking variance on both sides, we get

$$Var(e_{i,t}) = \widetilde{\beta}^2_{i,c} Var(e_{c,t}) + Var(\eta_{i,t}) \tag{11}$$

or using different notations:

$$\sigma^2_{i,T} = \widetilde{\beta}^2_{i,c}\sigma^2_{c,T} + \widetilde{\nu}_{i,T}, \qquad t \in T \tag{12}$$

where $\sigma_{i,T}$ is the realized volatility of $e_{i,t}$, $\sigma^2_{c,T}$ is the realized volatility of $e_{c,t}$, t is changed to lower frequency T due to data aggregation. $\widetilde{\nu}_{i,T}$ is their proposed idiosyncratic uncertainty. The main drawback of this decomposition is that it only controls for the first-moment dependency between $e_{i,t}$ and $\eta_{i,t}$ so $\widetilde{\beta}^2_{i,c}$ would not render orthogonality between $\sigma^2_{i,T}$ and $\widetilde{\nu}_{i,T}$. This limitation is mainly due to the fact that shocks are observed and measured on the first moment.

Alternatively, second-moment measurement FLU is derived from stochastic volatility model so I skip (11) and start directly from (12). Using a different notation, we have:

$$\sigma^2_{i,t} = c_i + \beta_{i,c}\sigma^2_{c,t} + \nu_{i,t} \text{ for each } i, \tag{13}$$

where $\sigma^2_{i,t}$ is FLU; c_i is the firm specific intercept that controls for level effects; $\sigma^2_{c,t}$ is market common uncertainty proxied by MU; $\nu_{i,t}$ is the disturbance with firm-specific variance. The resulting $\psi_{i,t} = c_i + \nu_{i,t}$ is the proposed firm Idiosyncratic Uncertainty (or IU, or $\psi_{i,t}$) that are orthogonal to macro volatilities (MU). [29] It is worth noting that $\beta_{i,c}$ from regression (13) is different from $\widetilde{\beta}^2_{i,c}$ from equation (12) in the sense that $\beta_{i,c}$ is estimated with all second-moment measurements while $\widetilde{\beta}_{i,c}$ is estimated with all first-moment shocks. Consequently, only $\beta_{i,c}$ returns desired second-moment independence between $\nu_{i,t}$ and $\sigma^2_{c,t}$. Based on equation (13), $1,865$ firm-level regressions are estimated for their corresponding idiosyncratic uncertainty (IU) series $\psi_{i,t}$.

Figure 15 illustrates the weighted average of IU together with MU. This new series should not be confused with MU because it captures the average level of firm-specific uncertainties that are orthogonal to MU. Despite the high similarity between both, several divergences might need attentions: Macro uncertainty is generally higher in the 80s, but idiosyncratic uncertainty outgrows MU in the early 90s and its dominance lasts for the entire great moderation. The gap again disappears prior to the 07-09 recession and both series closely follow each other thereafter. This relative movement implies a stronger negative impact of macro uncertainty as fast growing periods such as 90s and early

[29] An apparent drawback of equation (13) is that $\nu_{i,t}$ is not necessarily positive thus $\psi_{i,t} \geq 0$ is not guaranteed. However. such a problem is trivial because I only concern relative variations of idiosyncratic uncertainty rather than absolute variations. Also, all $\psi_{i,t}$ are adjusted by c_i so a negative $\psi_{i,t}$ is uncommon in the data.

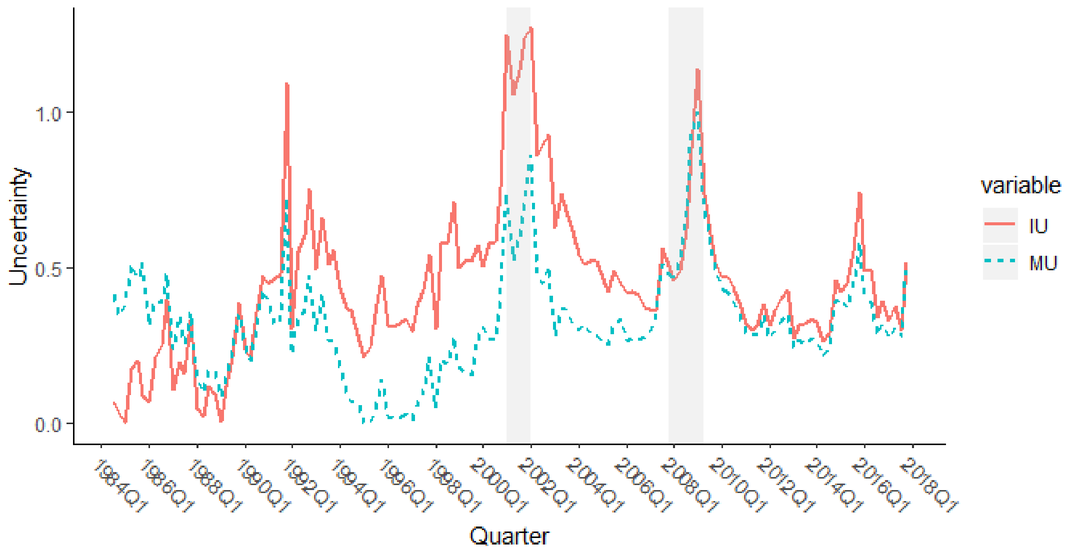
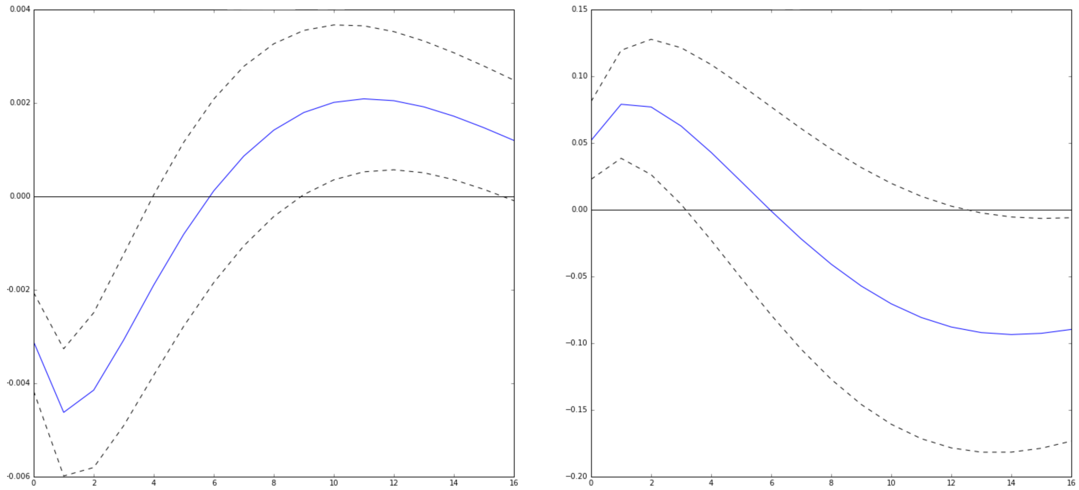


Figure 15: Average Firm-specific Uncertainty

IU: Average Firm-specific Uncertainty; MU: Macro Uncertainty Index

2000 all feature a low macro uncertainty and high firm-specific uncertainties.

To see the macro impact of IU, I use the weighted average of IU in the same VAR setup. Figure 16 illustrates the IRFs of industrial production and unemployment to shocks on IU. Similar to earlier results, IU generates a drop and rebound for industrial production, but only a drop in employment. While the dynamics is again significant and quite similar, the effect for both cases are now 20% smaller. This finding is consistent with the divergence between these two series. Similar empirical results might also be found in Ozturk and Sheng (2017), which suggests a smaller real effect of country-specific uncertainties comparing to the global uncertainty.



IRF of Industrial Production

IRF of Unemployment

Figure 16: Responses of Industrial Production and Unemployment to Average Idiosyncratic Uncertainty (IU) Shocks (90% Confidence Interval)

4.2 Uncertainty Effects at the Firm Level

Reasons why firms decide to change their tangible investments or financial portfolios facing rising uncertainty are diversified. VAR analysis draws a broad picture of firms' reactions, but it is unable to disentangle the effect associated with each component of uncertainty. In this section, I include both macro and idiosyncratic uncertainties in panel regressions to examine their respective impacts on firms' investing behaviors with a focus on the term structure.

Dependent variable includes short-term investment, long-term investment, and R&D. [30] Control variables include the standard first moment controls: Tobin's Q, the ratio of current cash flows to total assets (CFA), sales growth (S), and also variables that reflect companies' ability to borrow: leverage (L) and current ratio (CR).[31] Since the scale of many variables are quite different, I take logarithm of all level measurements to show their sensitivities to uncertainty shocks. Ratio measures maintain the original format. It is worth noting that the FLU measurements is short-term and

[30] Short-term investment refers to investments that are intended to be converted into cash within a relatively short period of time, usually within 1 year; Long-term investment has a horizon longer than a year; R&D is companies' expenses on new product developments and usually considered long-term as well.

[31] All data of firm characteristics are from Compustat database. Tobin's Q is defined as $\frac{Market\ value}{Total\ assets}$; Current cash flow is the cash flow from operating activities; Current ratio is defined as $\frac{Current\ asset}{Current\ liability}$; Following Barrero, Bloom, and Wright (2017), leverage is defined as $\frac{Assets}{Assets-(LT\ Debt-ST\ Debt)}$.

matches the short-term investment in the Compustat data. The exact specification is as follows:

$$
\begin{aligned}
log(Y_{i,t}) = \alpha + \beta_1 log(MU_t) + \beta_2 log(IU_{i,t}) + \beta_3 Q_{i,t} \\
+ \beta_4 CFA_{i,t} + \beta_5 log(L_{i,t}) + \beta_6 CR_{i,t} + \beta_7 log(S_{i,t}) + u_i + \epsilon_{i,t}, \quad (14)
\end{aligned}
$$

where Y is one of the three dependent variables listed above; MU is the macro uncertainty index in section 3.1; IU is the idiosyncratic uncertainty in section 4.1; u_i is panel random effects. I use the random effect model due to the small sample size compared to the firm population (See Green and Tukey (1960)).[32] MU and IU are orthogonal by construction so the regression is free of collinearity. Main regression results are shown in Table 3.

Panel (1) shows uncertainty effects on short-term investment. Both macro and idiosyncratic uncertainties have significant negative impacts on ST investment and the impact is stronger for macro uncertainty. These results are consistent with earlier VAR analysis where average IU generates about 20% less real impacts than MU. To my knowledge this is the first time economic effects of uncertainty are tested at both macro and micro levels with a consistent uncertainty measurement.

Panel (2) shows results for long-term investment. For this scenario, the effect of macro uncertainty becomes less significant which suggests a decreasing influence of short-term macro uncertainty on firms' long-run plans. However, a rise in IU significantly increases firms' long-term investments. Together with previous finding, I conclude that firms shift investments from short-term to long-term following a positive idiosyncratic uncertainty shock. The reason behind such a term structure change might be better understood by looking at results in panel (3).

Panel (3) shows uncertainty effects on R&D expenses for new products. Both MU and IU are significant but with opposite signs. The interpretation is that MU drags down market sentiment and makes companies hesitant to develop new products due to concerns over unstable overall demand. However, shocks on idiosyncratic uncertainty are likely associated with local supply or demand mismatch so increasing investment on new product and technology seems to be a reasonable response to address those local factors. Since new product development takes time and effort, the positive sign of IU in panel (2) can be largely explained by corresponding results in panel (3).

[32] All regressions are also tested with fixed effect model and all key results maintain.

Control variables such as Tobin's Q, leverage, and current ratio also show significant impacts on a company's investment structure. Sales growth, on the other hand, is positively associated with both short-term and long-term investments. Signs of these variables are in line with the literature.

To further examine whether these results associated with idiosyncratic uncertainty are robust, I use the first principal component of 5 macro volatilities in section 2.4 as an alternative measure of macro uncertainty in the regression and all results remain. (Table 4)

	(1)	(2)	(3)	(4)	(5)
$Y_{i,t}$	log ST Invest	log LT Invest	log R & D	log ST Invest	log ST Invest
Frequency	Quarter	Quarter	Quarter	Quarter	Quarter
log IU	-0.06***	0.03**	0.04***	-0.20***	-0.06***
log MU	-0.81***	-0.00	-0.15***	-0.76***	-0.81***
Tobin's Q	0.03**	-0.05***	-0.02***	0.02*	0.03**
Cash F/Asset	0.01	-0.00	-0.03***	0.01	0.01
log Leverage	-0.36***	0.09*	0.11***	-0.38***	-0.36***
Current Ratio	0.15***	-0.03***	-0.02***	0.15***	0.15***
log Sales	0.40***	0.53***	0.55***	0.39***	0.40***
log IU x High R				0.14***	
High R				-0.40***	
$(\log \text{IU})^2$					0.01*
Obs	11,491	13,046	9,336	11,491	11,491
R^2	0.26	0.33	0.46	0.27	0.27

***, ** and * indicate significance at the 1%, 5% and 10% level respectively.

Table 3: Micro Impacts of uncertainties on Investment and R&D

	(1)	(2)	(3)	(4)	(5)
$Y_{i,t}$	log ST Invest	log LT Invest	log R & D	log ST Invest	log ST Invest
Frequency	Quarter	Quarter	Quarter	Quarter	Quarter
log IU	-0.04***	0.03**	0.04***	-0.20***	-0.05**
Macro PC1	-0.07***	0.00	-0.01***	-0.07***	-0.08***
Tobin's Q	0.03**	-0.05***	-0.02***	0.02**	-0.01
Cash F/Asset	0.012	-0.00	-0.03**	0.01	0.04
log Leverage	-0.36***	0.08*	0.01***	-0.39***	-0.34***
Current Ratio	0.15***	-0.03***	-0.02***	0.15***	0.13***
log Sales	0.39***	0.53***	0.54***	0.38***	0.24***
log IU x High R				0.15***	
High R				-0.38***	
$(\log \text{IU})^2$					0.01
Obs	11,214	12,726	9,081	11,214	11,214
R^2	0.26	0.33	0.46	0.27	0.25

***, ** and * indicate significance at the 1%, 5% and 10% level respectively.

Table 4: Micro Impacts of Uncertainties on Investment and R&D (Alt. macro uncertainty measure)

5 A Composite Uncertainty Model

Theories concerning the economic impact of uncertainty mainly focus on two channels, both of which predict a negative effect (see Bloom (2014)). The first channel is referred to as “real options” theory. Bernanke (1983) argues that investors face a continuously varying set of investing options and would prefer to wait if they feel uncertain about the outcome. In other words, uncertainty increases the value of new information which makes “wait” a more compelling option despite its cost. While the “real option” theory works well for irreversible projects, it might not be appropriate for easily reversible projects or projects that cannot wait. A more universal uncertainty effect is related to the risk aversion and risk premia. On one hand, a strong sense of risk aversion leads to increased precautionary savings and a weak overall demand during uncertain time. With sticky prices, those extra savings cannot be converted to new investments so overall production slows down. On the other hand, uncertainty raises the risk premia due to a high probability of default, which makes financing projects more expensive. In this section, I use a model that incorporate Lucas island model (see Lucas (1973)) and CAPM (see Sharpe and Sharpe (1970)) to demonstrate a third channel of uncertainty effect – inefficient expectation. This model features both macro and idiosyncratic uncertainties in shaping investors’ expectations. The original Lucas model is used to explain the Phillips curve under imperfect information, but part of the mechanism is directly translated into investment decisions under uncertainty. [33] The goal of this model is to propose one of many possible channels that uncertainty affect economy but not to generalize such mechanisms.

The model starts with firm’s investment function (15) in a competitive economy. Firms are all small in size so their idiosyncratic uncertainty shocks have little macro-scale impact. I focus on investment return R and assume excess returns $(R_{i,t} - \beta_i R_{M,t})$ the motivations for new investments. I also assume a truncated condition similar to the discrete jump to zero market value of a “project” in McDonald and Siegel (1986). With perfect information, firms respond positively to their excess

[33] It is worth noting that information asymmetry, as the foundation of Lucas Model, also plays a key role in both “real option” and “risk premia” theories. The economic effect of uncertainty can, therefore, be better understood by combining all these channels.

profit margins.

$$y_{i,t} = \begin{cases} \alpha \exp(R_{i,t} - \beta_i R_{M,t}) & \text{if } \; R_{i,t} \geq 0 \\ 0 & \text{if } \; R_{i,t} < 0 \end{cases} \tag{15}$$

An exponential function is used to avoid negative investment when firm's return goes below market level. It also captures the "excitement" of small producers when facing large profit margins. Without loss of generality, I assume that the market return $R_{M,t}$ is strictly positive. β_i measures a firm's exposure to "swings" in the market condition and its value is exogenous. Lastly, I assume that firms temporarily halt production in current period if they observe negative returns. With perfect information, uncertainty plays no role. Firms observe market facts and make investment decisions accordingly.

The key mechanism of the Lucas model comes from the uncertainty caused by imperfect information. Firms might observe their own current investment return $R_{i,t}$, but they do not observe the current market average return $R_{M,t}$. Considering that macro data on market returns usually take more than one quarter to arrive, such an assumption is reasonable even without assuming producers are located on isolated islands. Firms understand that their expectations would deviate from the real value with an error under volatile market condition, or

$$R^e_{M,t} = R_{M,t} - \epsilon_{M,t}, \qquad \epsilon_{M,t} \sim \mathcal{N}(0,\, \sigma^2_{M,t}) \ , \tag{16}$$

where $R^e_{M,t}$ is the expected market average return; $\sigma_{M,t}$ is the market macro uncertainty that captures the predictability of the market. Based on CAPM, I assume that individual values deviate from the "effective" market average by a random amount, or

$$R_{i,t} = \beta_i R_{M,t} + \epsilon_{i,t}, \qquad \epsilon_{i,t} \sim \mathcal{N}(0,\, \sigma^2_{i,t}) \ , \tag{17}$$

where $\epsilon_{i,t}$ is the excess return of firm i; $\sigma_{i,t}$ measures the idiosyncratic uncertainty that exclusive to firm i. By model construction, $E(\epsilon_{M,t}\epsilon_{i,t}) = 0$.

Firms observe the difference between $R_{i,t}$ and $\beta_i R^e_{M,t}$, which is a composite errors $\epsilon_{i,t} + \beta_i \epsilon_{M,t}$. They also understand that only $\epsilon_{i,t}$ should be considered to determine the amount of new investment but is unobserved. Under rational expectations, they would use regression model

$$\epsilon_{i,t-n} = \theta_i(\epsilon_{i,t-n} + \beta_i \epsilon_{M,t-n}) + u_{i,t-n}, \quad (n = 1, 2, 3...) \tag{18}$$

on historical data to determine the proportion of ϵ_i. Hence, we have an empirical approximation of $\theta_i = \frac{\sigma_i^2}{\sigma_i^2+\beta_i^2\sigma_M^2}$, where σ_i and σ_M are proxies of idiosyncratic and macro uncertainty. Therefore, the conditional expectation of firm i's excess return is

$$E(\epsilon_{i,t}|I_{t-1}, R_{i,t}) = \frac{\sigma_i^2}{\sigma_i^2 + \beta_i^2\sigma_M^2}(R_{i,t} - \beta_i R^e_{M,t}) \ . \tag{19}$$

Substitute into equation (15), we get the conditional investment function of firm i under imperfect information:

$$y_{i,t} = \begin{cases} \alpha \exp(\frac{\sigma_i^2}{\sigma_i^2+\beta_i^2\sigma_M^2}(R_{i,t} - \beta_i R^e_{M,t})) & \text{if} \ \ R_{i,t} \geq 0 \\ 0 & \text{if} \ \ R_{i,t} < 0 \ , \end{cases} \tag{20}$$

where $\frac{\sigma_i^2}{\sigma_i^2+\beta_i^2\sigma_M^2}$ measures the sensitivity of firms in response to observed excess returns. Equation (20) shows the conditional investment on an observed gap $(R_{i,t} - \beta_i R^e_{M,t})$. For the unconditional case,

$$E(y_{i,t}) = \alpha P(R_{i,t} \geq 0) E\Big(\exp\big(\frac{\sigma_i^2(R_{i,t} - \beta_i R^e_{M,t})}{\sigma_i^2 + \beta_i^2\sigma_M^2}\big) \big| R_{i,t} \geq 0 \Big) \ , \tag{21}$$

where $P(R_{i,t} \geq 0)$ is the probability of $R_{i,t} \geq 0$. $R_{i,t}$ and $(R_i - \beta_i R^e_M)$ are distributed as $R_{i,t} \sim \mathcal{N}(R_{M,t}, \sigma^2_{i,t})$ and $(R_i - \beta_i R^e_M) \sim \mathcal{N}(0, \ \sigma_i^2 + \beta_i^2\sigma_M^2)$.

An analytical solution to equation (21) is not immediate due to the complexity in the joint distribution between $R_{i,t}$ and $R_{i,t} - \beta_i R^e_{M,t}$. A numeric solution through simulations is therefore used. Simulation parameters are set according to following rules: (1) The initial value of idiosyncratic uncertainty σ_i is much larger than that of market uncertainty σ_M; (2) The market average return R_M is set to be a constant at 10%; (3) The initial value of σ_i is equal to R_M so the "jump to zero" event has noticeable impacts on firms' average investments but not overwhelming. All these rules are consistent with empirical finding in this paper and as well as the literature.

5.1 The Partial Effect of σ_M

Figure 17 illustrates the simulation result based on varying σ_M with a fixed σ_i and corresponding $E(y_i)$ of 1 million iterations at each σ_M value. A clear negative trend between macro uncertainty and the firm's expected investment is recorded when σ_M increases from 0.05 to 0.30. A further rise in σ_M results in a flatter curve but no turning point is observed. The intuition is that a rise in macro uncertainty σ_M would decrease firms' sensitivity (θ_i) to observed excess returns and discourage firm's investment in general. In other words, observed excess returns become less credible if the macroeconomy is very noisy and firms hesitate to react to positive information. However, the same insensitivity also applies to negative excess returns, which would encourage firms to invest more. What leaves the total effect asymmetric and being unconditional negative is the truncated area associated with $R_{i,t} < 0$. As such a jump occur with a higher probability with a strict positive $R_{M,t}$ and the total probability of its occurrence is non-trivial (around 32% with $\sigma_i = R_M$), it generates imbalanced negative impacts on investments. Consequently, with a truncated production function, an increase in macro uncertainty would discourage investments in general and thus hurt the economy.

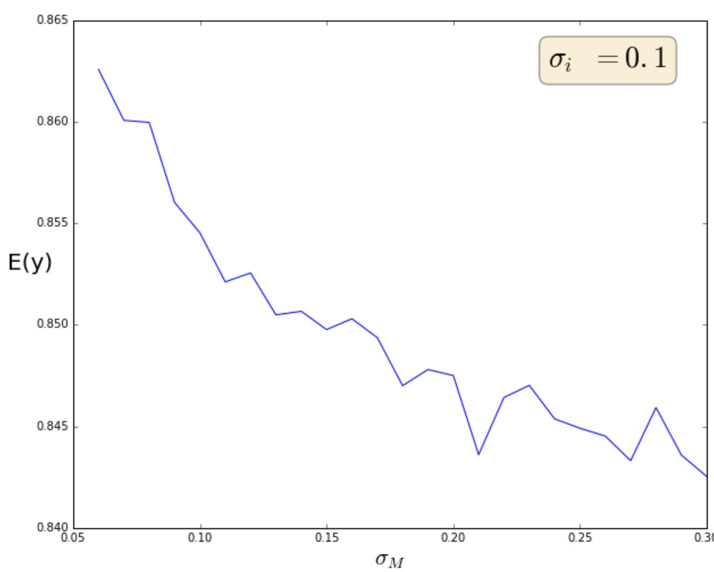


Figure 17: Simulation Result: Macro Uncertainty and Investment

This figure illustrates the relationship between y and σ_M in equation (2.7). By keeping σ_i constant, I simulate 1 million y at each σ_M value and plot the conditional mean $E(y|\sigma_M)$.

5.2 The Partial Effect of σ_i

From equation (2.7), we see that an increase in σ_i raises firms' sensitivity to excess returns and pushes its expected value further away from the center point conditional on either positive or negative excess returns. As a result, a rise of σ_i would only impose positive impacts on $E(y_i|R_{i,t} - \beta_i R^e_{M,t} \geq 0)$ and negative impacts on $E(y_i|R_{i,t} - \beta_i R^e_{M,t} < 0)$. The question is which effect dominates? The simulation result in Figure 18 shows that when σ_i rises from 0.1 to 0.25, there is a downward trend between σ_i and $E(y_i)$. However, further increasing σ_i from 0.25 to 0.35 unveils a turning point and the effect of σ_i eventually turns positive. The reason for such a convex path again comes down to the truncated area in the investment function. If σ_i is low, firms are unlikely to observe large excess return and thus unlikely to benefit big from higher sensitivity. Consequently, the relatively large negative impact of "jump to 0" dominates the positive effect from the increased sensitivity to positive excess returns. However, while the negative effect from a large σ_i value is bounded at 0, its positive effects are unbounded so further increasing σ_i would eventually turn the aggregate effect around. To illustrate my point, separate graphs for simulation points associated with only positive and negative draws of $(R_{i,t} - \beta_i R^e_{M,t})$ are shown in Figure 19. Idiosyncratic uncertainty positively affects firms' investments without a bound conditional on positive excess returns, but negatively affects investments with the lower bound 0 conditional on negative excess returns. Since firms have equal probability of facing negative and positive excess returns according to $(R_i - \beta_i R^e_M) \sim \mathcal{N}(0,\ \sigma_i^2 + \beta_i^2 \sigma_M^2)$, their combined effects follow a convex path. This mechanism is also known as "growth option" in Bar-Ilan and Strange (1996), who argue that uncertainty might positively affect investments if the potential prize of success is big. For this model, rising idiosyncratic uncertainty would increase the prize for getting a large positive draw. However, the loss of getting an unlucky negative draw is always bounded at 0.

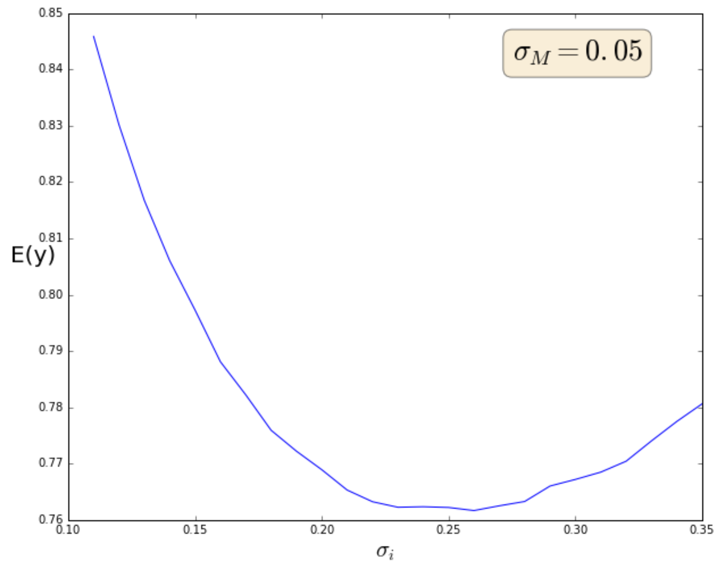


Figure 18: Simulation Result: Idiosyncratic Uncertainty and Investment

This figure illustrates the relationship between y and σ_i in equation (2.7). By keeping σ_M constant, I simulate 10000 y at each σ_i value and plot only the conditional mean of y ($E(y|\sigma_i)$).

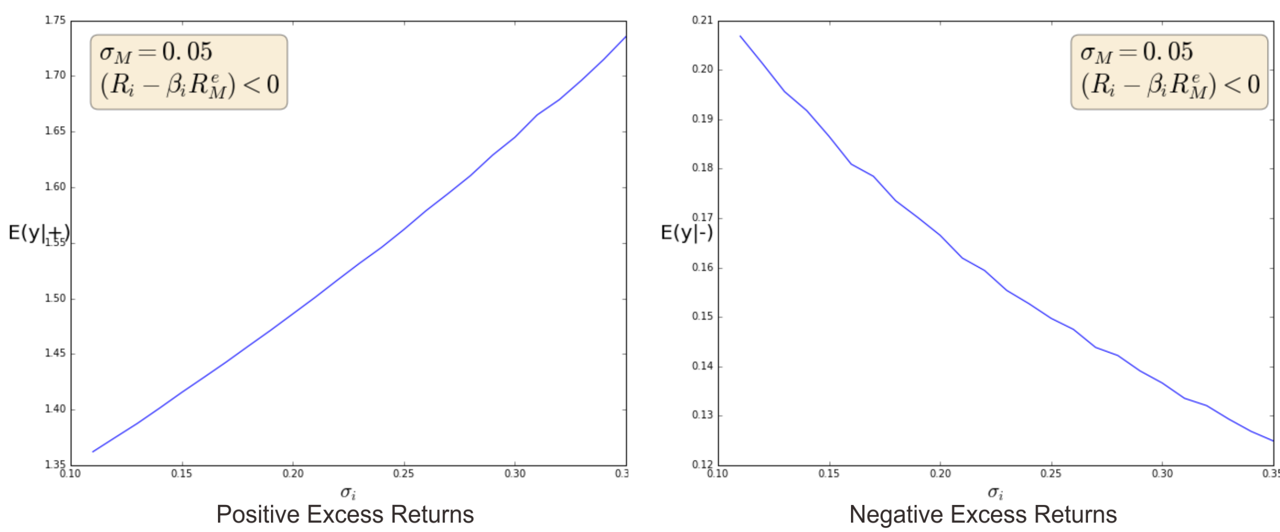


Figure 19: Simulation Result: Idiosyncratic Uncertainty and Investment (Separated by the Sign of Excess Return)

I divide all simulation points in Figure 2.2 into two groups based on the sign of $(R_i - \beta_i R_M^e)$, then plot the conditional mean of y ($E(y|\sigma_i)$) for each group similar to Figure 2.2.

5.3 Model Empirical Support

Model predictions are largely consistent with previous empirical results. The negative impact of macro uncertainty are captured in both the VAR analyses and the panel regression. For the "composite effects" of idiosyncratic uncertainty, figure 2 illustrates that fast-growing (high-profitability) companies such as Apple or Amazon show increasing idiosyncratic uncertainty over time. However, an average company does not have such a wide swing in investment returns so the overall effect of idiosyncratic uncertainty is negative as shown in table 3 column (1). To further test such composite relationship in model predictions, I include a dummy variable that takes the value of 1 if a firm's return is higher than the market average at t and interact it with IU in the panel regression. Results are shown in table 3 panel (4). The significant positive sign on the interaction term confirms the positive effect of IU on short-term investment for firms with higher returns. To capture the convex path between IU and firm investment in the simulation, I include the squared value of IU as a regressor and results are shown in Table 3 panel (5). The significant positive sign on IU^2 together with the significant negative sign on IU supports such a convex path.

6 Concluding Remarks

Uncertainty studies at granular level are largely limited by credible and compatible uncertainty measurements. To fill this gap, I propose a firm-level uncertainty measure based on the latent conditional volatility of forecast errors and apply it to the I/B/E/S database. The resulting Firm Level Uncertainty (FLU) tracks 1,916 U.S. public companies' uncertainties for over 34 years. The empirical study on macro drivers of FLU indicates that an average of 44% of FLU could be explained by 5 macro variables: financial and GDP uncertainty have generally large direct impacts; policy uncertainty turns out to be significant with an equal split between positive and negative effects; oil price volatility leads to countercyclical responses from many firms; exchange rate volatility has a much stronger impact on large firms which have extensive international businesses; the remaining drivers are mainly local factors that is hard to track down individually.

By carefully examining the size distribution of firms in the I/B/E/S sample, a Macro Uncertainty index (MU) is constructed using special weights. MU appears sensitive to economic recessions, financial crises, presidential elections, wars, and terrorist attacks. Both FLU and MU are carefully compared with other measures in the literature. MU has reasonable correlations with popular uncertainty indices such as VIX and EPU. FLU is closely linked to forecast disagreement and covers a much longer history with reasonable when compared to other firm-level measure such as Firm General Risk (FGR) in Hassan, Hollander, van Lent, and Tahoun (2019).

The macro effect of MU is tested in VAR models. Both industrial production and employment show drops following a MU shock, but fully recover and rebound a year later. More importantly, by taking advantage of FLU, I discover that the common uncertainty associated with the top 100 U.S. firms generates the economy-wide consequences which are not seen in smaller firms. The firm-level breakdown of uncertainty effects is examined in panel regressions. Results show that both macro and idiosyncratic uncertainty have significant negative impacts on short-term investment but the effect of macro uncertainty is much stronger. Similar results are also seen in VAR analysis using only macrodata. In comparison, idiosyncratic uncertainty changes firms' term structure of investments by shifting investment from short-term to long-term. One reason for such a shift is identified as increased spending on R&D when firms face idiosyncratic uncertainty shocks. This paper shows a consistent empirical results across micro and macro scopes.

A composite uncertainty model based on Lucas island model and CAPM is introduced to demonstrate on channel through which two types of uncertainty affect economic activities. The model predictions further uncover that the effect of idiosyncratic uncertainty depends on a firm's profitability and also the magnitude of shocks: firms of high profitability benefit from increasing idiosyncratic uncertainty while unprofitable firms suffer. On average, the relationship between idiosyncratic uncertainty and firm investment follows a convex path. All these relationships find empirical support in the data.

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